PCI BIG BEAM CONTEST

UNIVERSITY OF NOTRE DAME

Team Members:	Robert Devine David Terry Michael Brandes Jessica Orlando Ted Williams Nick Bette Megan McKeon Craig Davila
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BIG BEAM CONTEST 2015

5/6/15
Date

University of Notre Dame — Student Team (school name)
1 — Team Number
1/23/15 — Date of Casting

Basic information

1. Age of beam at testing (days) 35

2. Compressive cylinder tests*

Number tested: 3

Size of cylinders: 3x6

Average: 17,940 psi

3. Unit weight of concrete (pcf) 150

Slump (in.): 8”

Air content (%): 2%

Tensile strength (psi): 1,506

Circle one:
Split cylinder MOR beam

4. Pretest Calculations

a. Applied point load at midspan to cause cracking (kip) 22.2

b. Maximum applied point load at midspan (kip) 36.7

c. Maximum anticipated deflection due to applied load only (in.)
5.35

Pretest calculations MUST be completed before testing.

*International entries may substitute the appropriate compressive strength test for their country.

Test summary forms must be included with the final report, due June 16, 2015

Judging Criteria

Teams MUST fill in these values.

a. Actual maximum applied load (kip) 38.5

b. Measured cracking load (kip)‡ 22.5

c. Cost (dollars) 131.35

d. Weight (lb) 982.8

e. Largest measured deflection (in.) 5.44

f. Most accurate calculations .d

(a) Absolute value of (maximum applied load – calculated applied load) / calculated applied load
0.049

(b) Absolute value of (maximum measured deflection – calculated deflection) / calculated deflection
0.017

(c) Absolute value of (measured cracking load – calculated cracking load) / calculated cracking load
0.014

Total of three absolute values (a + b + c) = 0.080

‡Measured cracking load is found from the “bend-over” point in the load/deflection curve. Provide load/deflection curve in report.

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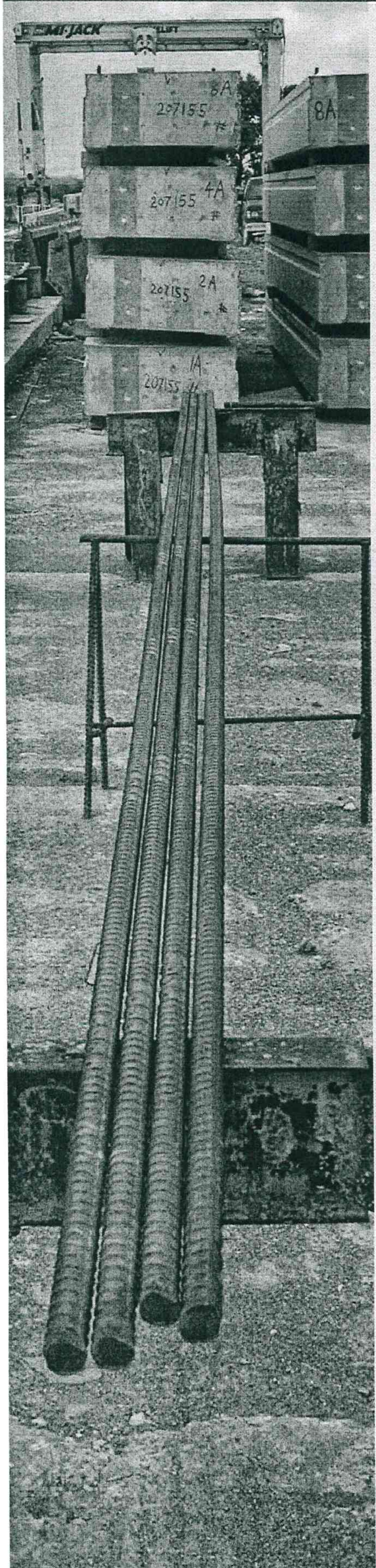
PCI BIG BEAM COMPETITION – 2015



CERTIFICATION

STRES CORE INC
As a representative of (name of Producer Member or sponsoring organization)

NOTRE DAME
Sponsoring (name of school and team number)

I certify that:

- The big beam submitted by this team was fabricated and tested within the contest period.
- The calculations of predicted cracking load, maximum load, and deflection were done prior to testing of the beam.
- The students were chiefly responsible for the design.
- The students participated in the fabrication to the extent that was prudent and safe.
- The submitted test results are, to the best of my knowledge, correct, and the video submitted is of the actual test.

Certified by:

John S Reihl
Signature

JOHN S. REIHL
Name (please print)

5-8-15
Date

THIS CERTIFICATION MUST BE PART OF THE FINAL REPORT

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Table of Contents

1. Drawings and Geometric Design

Several cross-sections were considered before a pretensioned T-beam design was finalized. Figure 1 shows the dimensions of the final cross section, while Figure 2 calls out the final stirrup spacing. A T-shape was chosen because of its ability to efficiently handle the loading conditions. The main goal in choosing the beam dimensions was to minimize both the cross-sectional and surface area in order to reduce the concrete and formwork costs, respectively. A beam with a wide flange and slender web allowed for the use of a large amount of concrete in the compression zone under ultimate loads while minimizing the amount of concrete in the tension zone.

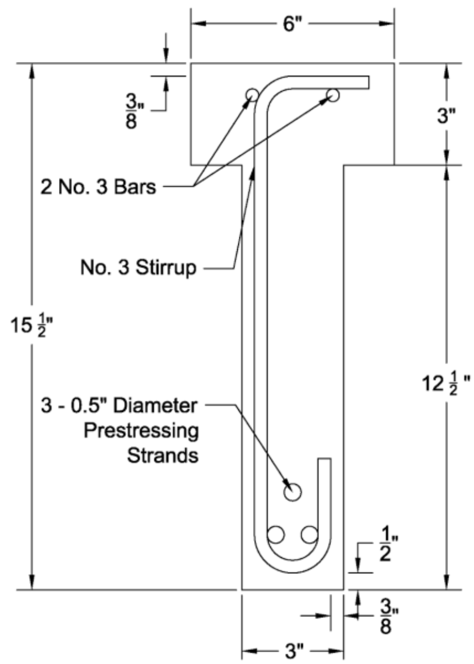

Figure 1: Beam Cross-Section

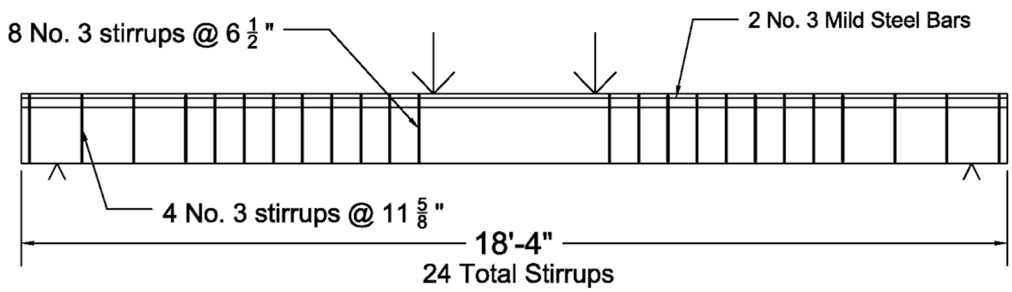

Figure 2: Beam Elevation

In order to design the most optimal beam, an Excel spreadsheet was created to calculate the area, moment of inertia, centroid, section modulus, and other section properties. After several iterations, the most optimal beam design for both performance and cost efficiency was a uniform flange of 6 inches wide by 3 inches thick, a uniform web of 3 inches thick, and a constant beam height of 15.5 inches. The most critical component of the formwork cost was the height of the beam, so a relatively shallow beam (15.5 inches) was used to reduce costs. In order to compensate for the small depth of the beam, it was necessary to use three prestressing strands. It was determined that the cost of this beam was much smaller than the cost of a deeper beam with two strands. The total length of the beam was 18 feet 4 inches after an extra length of 8 inches were added to both ends to ensure stability beyond the supports. Additional information on beam geometric design details can be found in Appendix A.

2. Concrete Mix Design

In the initial phase of the concrete mix design, it was uncertain whether a high-strength mix or normal strength mix would be ideal. In accordance with PCI Competition rules, any concrete mix with a compressive strength over 10,000 psi was considered a high-strength mix and incurred an additional cost over normal strength mixes. At first, the goal was to design a concrete mix that reached a compressive strength of just under the 10,000 psi threshold, maximizing the strength that was considered a normal mix. However, it was soon found that a high-strength mix would provide the ability to design a shallower beam. Due to the high cost of formwork and the relatively small cost difference between high and normal concrete strength, the team moved on to high strength mixes. The concrete compressive strength that the beam with the cross-sectional properties in Figure 1 required was 17,000 psi, which became the goal of the concrete mix design effort.

A total of 6 trial mix designs were completed, with the constituent components and properties listed in Table 1. The corresponding strengths of each mix are summarized as well in Table 2. Knowing that low water/cement ratios produce higher strength concrete, the design philosophy was to use the lowest ratio while having a functionally workable mix. Mixes 1-3 did not meet this second criteria, as their workability was so poor that test cylinders had large air pockets, resulting in lower strengths. The water/cement ratio was increased for mixes 4-6, which created more workable mixes that consolidated well enough to take advantage of the strength gain due to low ratios. The amount of High Range Water Reducer was kept constant between the different trials. Two different coarse aggregates were considered through the process. Originally, INDOT #11 limestone was used in early mix design trials but in the final mix, 3/8 inch nominal granite was used as the coarse aggregate to reach the required concrete strength. Since granite is not readily available in Indiana, it was ordered from Kafka Granite, located in Wisconsin. The team ordered two tons of Platinum Granite from Kafka Granite LLC, which was then shipped and stored at Ozinga Ready Mix Concrete - the company that poured the final mix for the casting of the beam. Although not common to Indiana, granite is a practical coarse aggregate in states, such as Wisconsin, in which it is readily available.

Table 1: Laboratory Trial Concrete Mix Designs

Material	*Mix 1*	*Mix 2*	*Mix 3*	*Mix 4*	*Mix 5*	*Mix 6*
Portland Cement Type I (lbs/yd^3)	1200	1200	1200	950	950	950
INDOT #11 Limestone (lbs/yd^3)	1830	1830	1830	1830	1830	-
Kafka Platinum Granite (lbs/yd^3)	-	-	-	-	-	1850
INDOT #23 Sand (lbs/yd^3)	1460	1460	1300	1300	1300	1300
Silica Fume (lbs/yd^3)	280	280	90	90	90	90
Fly Ash (lbs/yd^3)	50	50	50	50	50	50
Water (lbs/yd^3)	260	260	280	280	255	255
Water/Cement	0.17	0.17	0.21	0.26	0.23	0.23
High Range Water Reducer (oz/cwt)	36	36	36	36	36	36

After trial batching each mix in the laboratory, three 3 x 6 inch concrete cylinders were made to determine the average compressive strength and a standard slump test was performed to determine workability. All batching and testing of the trial mixes were done in the University of Notre Dame Structural Systems Laboratory and adhered to ASTM C39/C39M and ASTM C143/C143M guidelines. Compression tests of the 3 x 6 inch cylinders were initially conducted at three days. As the mix design became more finalized, cylinders were tested at seven days to determine the seven-day compressive strength as seen in Table 2. This test was important as it allowed a rough estimate of the twenty-eight day compressive strength to be estimated in advance. Twenty-eight day tests of the final mix design resulted in a compressive strength of 16,000 psi. Even though this was less than the target design strength of 17,000 psi, the concrete from Ozinga Ready Mix Concrete to be used in the casting of the prestressed beam was expected to be slightly stronger than the laboratory mixes. Factors such as the larger batch size and a truck mixer would decrease variability and mix the concrete more thoroughly compared to a laboratory mixer, increasing the final twenty-eight day compressive strength.

Mix 6 from Table 1 was ultimately chosen due to expected higher strengths and smaller aggregate sizes which our narrow beam required. This mix design was batched and delivered by Ozinga Ready Mix Concrete for the casting of the prestressed beam. In addition to the beam, twelve 3 x 6 inch cylinders and three modulus of rupture (MOR) beams were also cast and stored under the same environmental conditions as the beam. The compression strength of the concrete at seven, fourteen, twenty-eight, and beam test day (day 35) can be seen in Table 3. These cylinders were also weighed and it was determined that their average unit weight was 150 pcf. As expected, the seven day strength of the ready mix concrete (13,350) was greater than that of the laboratory mix (10,205). Also, the twenty-eight day strength of the ready mix concrete (17,083 psi) satisfied the target design concrete strength (17,000 psi). The MOR tests were only conducted at twenty-eight days, and resulted in a tensile strength of 1,506 psi. The slump was greater than 8 inches, which ensured that the concrete would consolidate with minimal vibration. Excessive vibration not only could cause the aggregate to sink in the form, but could also potentially shift the rebar cage, compromising the accurate fabrication of the beam. Finally, the air content of the mix, 2.0%, was provided on casting day by Ozinga from their mix ticket.

Table 2: Average 7-Day Strength of Laboratory Mixes	
Mix No.	*Compression Strength (psi)*
1	5,384
2	5,709
3	6,832
4	10,679
5	10,633
6	10,205

Table 3: Concrete Properties on Testing Day of Beam	
Slump (inches)	> 8
Tensile Strength, MOR (psi)	1,506
Air Content	2.00%
Unit Weight (pcf)	150
7 Day Compressive Strength (psi)	13,350
14 Day Compressive Strength (psi)	15,728
28 Day Compressive Strength (psi)	17,083
Beam Test Day Compressive Strength (psi)	17,940

3. Structural Design

Allowable Stress Design

The maximum service moment due to the self-weight of the beam and the two prescribed 10-kip superimposed point loads (total load of 20 kips), M_s, was found to be 865 kip-inches. Four allowable stress equations were used to ensure stresses in both the top and bottom of the beam cross section would be within limits at prestress transfer as well as under full service load. The resulting eccentricities were as follows: e_1 – tension stress at beam top at prestress transfer; e_2 – compression stress at beam bottom at prestress transfer; e_3 – compression stress at beam top under full service loads; and e_4 – tension stress at beam bottom under full service load. The maximum allowable stresses for these four conditions were determined from ACI 318-11. To determine the initial prestress force and the eccentricity of the prestressing strands, the four allowable stress equations were plotted on a Magnel Diagram as shown in Appendix B.

ACI 318-11 permits the allowable stress limit for e_1 to be violated if an adequate amount of mild steel is placed at the top of the beam. To take advantage of this provision, two No. 3 bars (determined from the design calculations shown in Appendix B) were placed in the top flange to resist the tension force in the flange upon transfer of the pre-stressing force to the concrete. For e_4, ACI 318-11 provides an allowable concrete tensile stress of $7.5\sqrt{f_c'}$. A slightly more conservative value of $6.5\sqrt{f_c'}$ was used to ensure cracking would not occur under the prescribed service loads. The maximum eccentricity of the prestressing strands, e_{max}, was also limited by the physical depth of the beam. The most efficient and practical governing design point on the Magnel diagram was at the intersection of e_4 and e_{max}. The e_2 limit for the compressive stress at the bottom of the beam at prestress transfer, and the e_3 limit for the compressive stress at the top of the beam under service loads did not govern this design. The final allowable stress design of the beam required three 0.5 in. diameter 270 ksi low-relaxation prestressing strands placed at $e = e_{max} = 6.76$ inches from the beam centroid, with an initial pre-tensioning force of $P_i = 85.6$ kips. The slenderness of the web required the strands to be placed in a triangular layout shown in Figure 1. All calculations for the allowable stress design can be found in Figure B1 in Appendix B.

Ultimate Flexural Strength Design

The competition rules required the ultimate strength of the beam to be between 32 and 40 kips of total load applied at the two specified points. To satisfy this requirement, the beam was designed to fail due to flexure under an ultimate load of 36 kips. Ultimate strength calculations were done to determine if any mild steel reinforcement was needed for tensile strength at the bottom of the beam in addition to the three prestressing strands. From these calculations, it was found that the three prestressing strands provided the necessary ultimate strength, so no additional mild steel was necessary. The calculations for the ultimate strength design of the beam can be found in Appendix B.

Shear Design

Shear design of the beam was based on Chapter 11 of ACI 318-11. The contribution of concrete to the shear resistance, V_c, was large due to the use of high strength concrete. A single-leg stirrup was chosen to accommodate for the small web width and also because this configuration provided the most efficient design while satisfying all of the minimum shear reinforcement requirements of the code. Figure 1 shows that each stirrup was made from a No. 3 bar bent at a constant radius around the prestressing strands. The bar extended up on one side of the web to the mild steel bars in the top flange. During the placement of the stirrups, the hook orientation was alternated in an attempt to prevent eccentric shear resistance along the length of the beam.

The shear design calculations are given in detail in Appendix B. The ultimate shear force diagram, V_u, was determined based on the self-weight and maximum loading from the ultimate flexural strength of the beam. The design shear strength contribution from the stirrups, $øV_s$ was determined from the difference between V_u and $øV_c$, where the capacity reduction factor was taken as ø=0.75. The calculations showed that the maximum stirrup spacing of 11.625 inches - allowed by Section 11.4.5.1 of ACI 318 - governed the design within a distance of 28 inches from each support. At a distance of 28.75 inches from each support, the required stirrup spacing dropped to a minimum value of just above 6.5 inches. The stirrup spacing was therefore kept constant at 6.5 inches beyond the last stirrup spacing at 11.625 inches. Finally, between the two superimposed load points, the ultimate shear force, $V_{u,}$ was less than $øV_c/2$ which resulted in stirrups not being required and therefore not used in this region.

Longitudinal Strand Reinforcement Design

As previously mentioned above in the ultimate strength design, the e_1 limitation was waived as the team would instead use rebar to reinforce the top flange for tensile stresses at pretress transfer. To begin this design, two No. 3 bars were assumed for the design using prior knowledge from beam designs in years past. The tensile stress was assumed to have a triangular stress distribution from the top of the flange to the neutral axis, which was found to be 3.2 inches from the top fiber and slightly within the web of the beam. From this stress distribution, a force resultant was found to be 13.1 kips. Assuming yielding of the reinforcing bars and no tensile resistance from the concrete, it was found that two No. 3 bars would sufficiently prevent any significant cracking along the top fiber. Calculations for this process can be reviewed in Appendix B.

4. Beam Fabrication

Formwork Design and Construction

All safety protocols established by the University of Notre Dame and the Structural Systems Laboratory were followed during the fabrication of the beam. The construction team members worked with the structural design members to determine the formwork details. This design-build interaction allowed the efficiency and cost of the formwork to be considered as part of the structural design, resulting in an optimal final solution. After the shape and dimensions of the beam were determined, this information was input into CAD to produce the formwork virtually. Referencing the new CAD drawing, necessary quantities for the formwork were finalized and ordered. The cross section of the final formwork design can be seen in Figure 3.

Team Members work on the flange overhang of the formwork

To save on cost of supplies, the construction of the formwork involved the use of standard 2 inch by 4 inch lumber and standard ¾ inch plywood. The total amount of lumber and the necessary cuts to complete the formwork was determined in CAD. Since standard ¾ inch plywood is actually less than ¾ inches thick, shims were placed between the sheets of plywood where necessary to result in the desired formwork dimensions. The sides and bottom of the formwork were cut from 4 feet by 8 feet plywood sheets. Then, 2 inch by 4 inch lumber was used to reinforce the sides of the formwork at a spacing of 16 inches, as seen in Figure 4. To ensure a consistent width, the top of the formwork was braced by 2 inch by 4 inch lumber spaced at 2 feet.

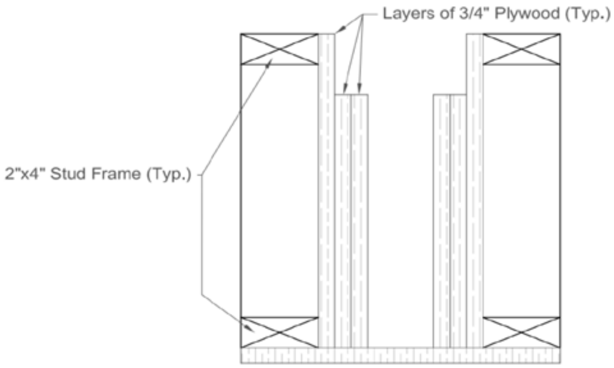

Figure 3: Formwork Cross Section

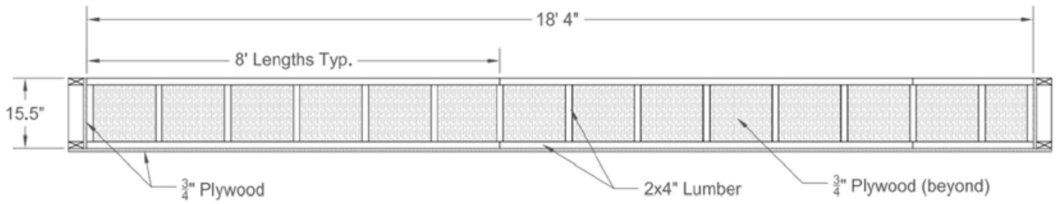

Figure 4: Longitudinal Formwork Layout

Rebar Cage Assembly and Placement

The rebar cage was assembled directly inside the formwork using single leg stirrups and two longitudinal No. 3 bars in the flange as shown in Figure 1. The stirrups were placed inside the formwork with the hooks in an alternating pattern and then the two longitudinal mild steel bars were passed through and wire-tied to each stirrup. Concrete cover around the reinforcement was controlled using shims. On the day of concrete placement, three 0.5-inch steel prestressing stands were threaded through the bottom hooks of the stirrups. Great care was taken to ensure that strands were not entangled. Then, the final positioning and spacing of the stirrups and top bars were adjusted and fixed to conform to the design requirements, mostly ensuring no stirrups were off their vertical axis.

Bending of Stirrups

Beam Casting and Prestressing

The beam was cast on Friday January 23, 2015 on a heated prestressing bed at StresCore, Inc., South Bend, which is a PCI certified plant specialized in hollow core panels. Members of the team all wore safety goggles, hard hats, and hard-toe closed-shoes during the entire operation, and followed all safety protocols at StresCore, Inc. After the final adjustment of the reinforcement cage, the three strands were tensioned using the bulkheads at each end of the bed. The concrete was delivered directly from the Ozinga Ready Mix Concrete truck into the formwork in one continuous operation. The students used vibration rods to ensure no air pocketing issues, while also being sure not to over-vibrate. This operation was followed with trowels to finish the top surface of the beam, especially in locations where load would be applied during testing. The beam was then left on the heated bed over the weekend to allow time for the concrete to set. On Monday January 26, 2015, the team viewed (from a safe distance) the cutting of the pretensioned strands by the StresCore staff. Each strand was cut at both ends of the beam simultaneously, successfully transferring the prestress force into the concrete. The beam was then moved to an outdoor storage area at StresCore, Inc. for two weeks, before being transported to Notre Dame and stored indoors until test day.

5. Pretest Predictions

Cracking Moment

A pre-test prediction for the cracking moment of the beam was made by reversing the design process and updating the geometric and material properties from design to the as-built properties for the beam. For the cracking moment, the largest change from design to pre-test prediction was the tensile strength of the concrete. In the allowable stress design, a concrete tensile strength of $6.5\sqrt{f_c'}$ = 847 psi was conservatively assumed, as compared to the well-known ACI equation of $7.5\sqrt{f_c'}$. For the pre-test analysis, the average measured tensile strength of 1,506 psi from the modulus of rupture (MOR) tests was used. With this increased strength for the concrete tension strength, the analysis predicted a cracking moment of 73.6 k-ft, with an associated applied load of 22.2 kips. The calculations associated with all pretest predictions can be found in Appendix C.

Ultimate Strength

The ultimate strength of the beam was determined by reversing the design procedure in a similar fashion. The concrete compression strength was updated using the average measured strength from the cylinders made during the casting of the beam and stored in similar conditions as the beam. The two No. 3 mild steel bars placed inside the flange were included as compression reinforcement in the pre-test prediction, even though this steel was ignored in the ultimate strength design of the beam. Through an iterative process to determine the strains and stresses in the mild steel and prestressing strands using equilibrium, kinematics, and constitutive relationships, together with an assumed useable concrete strain of 0.003, the maximum moment strength of the beam was found to be 130.6 k-ft, with an associated applied ultimate load of 36.7 kips.

Maximum Deflection

The maximum deflection of the beam at mid-span was estimated using moment-area theorems. These theorems require the nonlinear moment versus curvature behavior of the beam cross section to be determined and the curvature diagram along the length of the beam under the predicted ultimate load to be drawn. The nonlinear moment-curvature behavior of the beam was determined using a structural analysis program, Response 2000 (Bentz et al. 2000). As-built material and geometric properties of the beam were again used for this purpose. The beam was discretized along its length and the moment at each discretization point was determined corresponding to the predicted ultimate moment at the midspan. Then, the curvature at each discretization point was determined from the Response 2000 nonlinear moment-curvature diagram for the beam section. The curvature diagram was assumed linear between the discretization points. Therefore, as the curvature was more non-linear near the beam midspan, a smaller distance between each discretization point was used. Through this process, a deflection of 5.35 inches was predicted after the initial camber of 0.94 inches was added to the calculated deflection of 4.41 inches from level position.

6. Testing of Beam

Testing of the beam occurred on February 27, 2015, at an age of 35 days after casting. The beam was simply supported with the bottom about eight inches off the ground using steel wedges. The wedges were 17 feet apart, providing the stipulated clear span for the competition. Four small screw jacks were used on the sides of the beam and underneath the flanges as safety to stabilize the beam before testing. These jacks were removed before the testing of the beam. The superimposed load was applied by a hydraulic jack with an integrated load cell. The jack applied the load directly to a stiff steel spreader I-beam, which transferred the load to the concrete beam via transverse steel bars three feet apart. The applied total load was measured by the load cell in the hydraulic jack, and the mid-span deflection of the beam was measured using two linear potentiometers, one attached on each side of the beam below the flange. This four-point bending set up is shown in Figure 5 below.

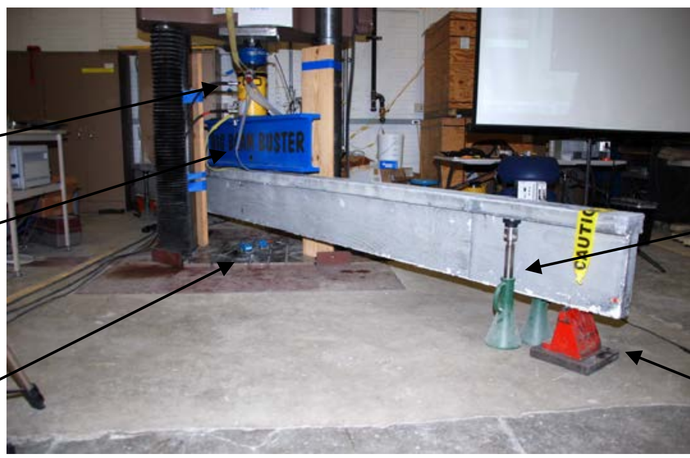

Figure 5: Four Point Bending Test Set Up

All safety protocols established by the University of Notre Dame and the Structural Systems Laboratory were followed during the testing of the beam. Hard hats, safety glasses, and hard-toe closed-shoes were worn, and all observers stood at a safe distance until the conclusion of the test.

Cracking Load

The beam exhibited excellent behavior over the course of the test. Beginning from an initial camber of about 0.94 inches, the beam initially deflected elastically. The cracking load from the bend-over point on the load deflection curve was found to be about 22.5 kips, as seen in Figures 6 and 7. More clearly in Figure 7, initiation of cracking occurs as the load deflection curve deviates from the straight line segment in the figure, with the corresponding load of 22.5 kips, satisfying the competition's service loading requirement of 20 kips.

Peak Load and Ultimate Failure

As the load was increased, flexural cracks could be observed between the load points. Additionally, flexural-shear cracks developed outside the loading points. As the beam reached its ultimate load, it exhibited desirable flexure-dominated behavior. Since the critical moment was in the mid-span, the region between the applied loads responded like a plastic hinge, resulting in a sharp angle of bending at the center. Shear failure did not occur and the prestressing steel strands yielded, resulting in a ductile behavior as intended. Final failure occurred at a load of 38.5 kips, when the concrete flange (and upper part of web) crushed just inside one of the loading points. This met the competition's required ultimate strength range of 32 to 40 kips.

Maximum Deflection

Under a service load of 20 kips, the beam exhibited a deflection of 0.47 inches, which is approximately 0.2% of the span length. This value is under the ASCE 7-10 total load deflection limit (L/240) of 0.85 inches. At failure, the total deflection of the beam was 5.44 inches, which was close to the predicted value of 5.35 inches.

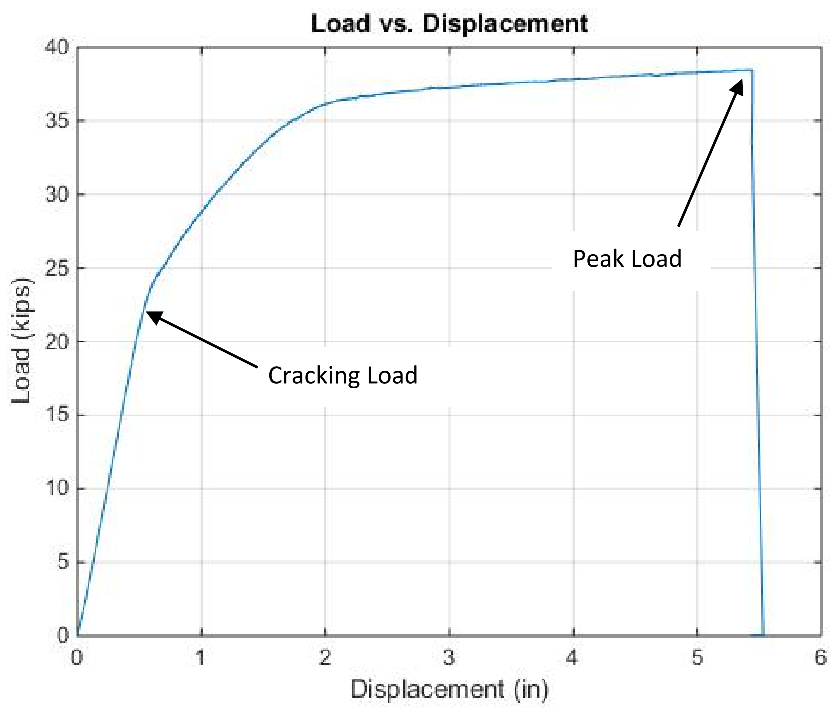

Figure 6: Load-Displacement Curve

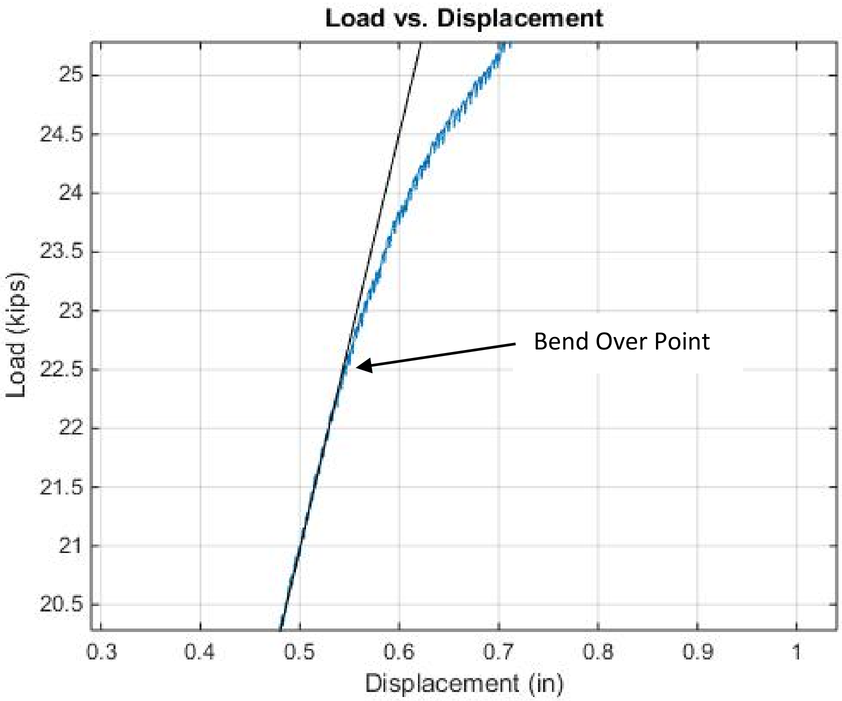

Figure 7: Bend Over Point

Errors in Predicted versus Measured Behavior

The errors in the prediction of the cracking load, ultimate load, and maximum deflection as compared with the corresponding measured values were 1.4%, 4.9%, and 1.7%, respectively, as shown in Appendix C. The total combined error from these three response quantities was 8.0%. Table 4 summarizes our design results, predicted results, and actual results as compared to the PCI competition requirements.

Table 4: Summary of Results

	PCI Competition Requirement	Designed Value	Predicted Value	Actual Value
Cracking Strength (kips)	20	20	22.2	22.5
Ultimate Strength (kips)	32-40	36	36.7	38.5
Deflection (inches)	-	-	5.35	5.44

7. Recommendations

Although the design of the beam was very effective and resulted in the intended performance, the constructability of the beam could be improved. The narrow web made construction difficult, especially with respect to the reinforcement cage. The stirrups were bent into single legs to achieve an efficient design for shear, which made the reinforcement cage unstable during handling and transportation. Also, to ensure that the specified cracking load is satisfied, it may be better to target a higher service load rather than assume a lower concrete tensile strength in design. Finally, making contact with the prestressing company should be made earlier in the design process to prevent any conflicts with strand layout or placement.

8. References

Bentz, E. and Collins, M., "Response-2000: Reinforced Concrete Sectional Analysis using the Modified Compression Field Theory," V1.0.0, University of Toronto, 2000.

ACI 318, "Building Code Requirements for Structural Concrete and Commentary," American Concrete Institute, Farmington Hills, MI, 2011.

ASTM Standard C39/C39M-14a, 2003, "Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens," ASTM International, West Conshohocken, PA, 2003, DOI: 10.1520/C0033-03, www.astm.org.

9. Appendices

Appendix A: General Beam Information
Appendix B: Design Calculations
Appendix C: Pretest Calculations
Appendix D: Additional Photographs

APPENDIX A

GENERAL BEAM INFORMATION

A1. Design Material Properties

Design Compressive Strength of Concrete	$f_c' = 17\ ksi$
Design Tensile Strength of Concrete	$f_r = 6.5\sqrt{f_c'} = 0.848\ ksi$
Ultimate Strength of Prestressing Strand	$f_{pu} = 270\ ksi$
Prestressing Strand Initial Stress	$f_{ps} = 0.7 * f_{pu} = 189\ ksi$
Total Area of Prestressing Strand	$A_{ps} = 3 * 0.153 = 0.459\ in^2$
Total Area of Top Mild Steel	$A_s = 0.22\ in^2$
Unit Weight of Concrete	$\gamma = 150\ pcf$
Concrete Modulus of Elasticity	$E_c = 57000\sqrt{f_c'} = 7{,}432\ ksi$
Prestressing Steel Modulus of Elasticity	$E_s = 28{,}500\ ksi$

A2. Geometric Properties

Beam Height	$h = 15.5\ in$
Flange Height	$h_f = 3\ in$
Flange Width	$b_f = 6\ in$
Web Width	$b_w = 3\ in$
Span Length	$L = 17\ ft$
Total Length	$L_t = 18\ ft\ 4\ in$
Moment of Inertia	$I = 1232.3\ in^4$
Distance to Centroid from Top Fiber	$c_1 = 6.74\ in$
Distance to Centroid from Bottom Fiber	$c_2 = 8.76\ in$
Top Section Modulus	$S_1 = \frac{I}{c_1} = 182.9\ in^3$
Bottom Section Modulus	$S_2 = \frac{I}{c_2} = 140.6\ in^3$

A3. Cost Calculation

Cross Sectional Area	$A = 55.5\ in^2$
Formwork Surface Area	$SA = 57.3\ ft^2$
Volume of Beam	$V = 6.552\ ft^3 = 0.243\ yd^3$
Weight of Beam	$W = 982.81\ lbs$
Weight of Mild Steel	$W_{ms} = 31.35\ lbs$
Cost of Concrete	$C_c = 120(\frac{\$}{CY}) * V(CY) = \$29.12$
Cost of Prestressing Steel	$C_{ps} = 0.30\left(\frac{\$}{ft}\right) * 3 * L_t(ft) = \$16.50$
Cost of Mild Steel	$C_{ms} = 0.45\left(\frac{\$}{lb}\right) * W_{ms}(lb) = \$14.11$
Cost of Formwork	$C_f = 1.25\left(\frac{\$}{ft^2}\right) * SA(ft^2) = \$71.62$
Total Cost of Beam	$C = \$131.35$

APPENDIX B

DESIGN CALCULATIONS

B1. Allowable Stress Design

Cement Factor	$\alpha = 4.0$
Type I Cement Moist Cured Factor	$\beta = 0.85$
Transfer Time	$t_{transfer} = 3\ days$
Concrete Strength at Pre-Stress Transfer	$f'_{ci} = \frac{t_{transfer} * f'_c}{\alpha + \beta * t_{transfer}} = 7.78\ ksi$
3 Day Test Concrete Strength	$f'_{c3} = 8.5\ ksi*$ (Note: *Used test data rather than theoretical)

1. Initial Tension Stress Limit

$$\sigma_{ti} = 6\sqrt{f'_{c3} * 1000} = 0.553\ ksi$$

2. Initial Compression Stress Limit

$$\sigma_{ci} = -0.7 * f'_{c3} = -5.95\ ksi$$

3. Service Uncracked Tension Stress Limit

$$\sigma_{ts} = f_r = 0.848\ ksi$$

4. Service Compression Stress Limit

$$\sigma_{cs} = -0.6 f'_c = -10.2\ ksi$$

Magnel Diagram Calculations

Initial Prestressing Force	$P_i = A_{ps} * f_{ps} = 86.75\ kips$
Prestressing Force after Losses	$P_e = 0.925 * P_i = 80.24\ kips$
Beam Self-Weight Load	$w_o = \gamma * \frac{A}{144} = 57.8\ lb/ft$
Beam Self-Weight Moment	$M_o = \frac{(w_o * L^2)}{8000} = 25.06\ k - in$
Service Load	$P_s = 20\ kips$
Beam Service Moment	$M_s = \frac{P_s * 84\ in}{2} + M_o = 865.06\ k - in = 72.08\ k - ft$

Magnel Equations

1. Eccentricity Associated with Tensile Stress at Top Fiber at Prestressing

$$e_1 = \frac{S_1}{A} + \frac{\sigma_{ti} * S_1}{P_i}$$

2. Eccentricity Associated with Compressive Stress at Bottom Fiber at Prestressing

$$e_2 = -\frac{S_2}{A} + \frac{-\sigma_{ci} * S_2}{P_i}$$

3. Eccentricity Associated with Compressive Stress at Top Fiber at Service Load

$$e_3 = \frac{S_1}{A} + \frac{\sigma_{cs} * S_1 + M_s}{P_e}$$

4. Eccentricity Associated with Tensile Stress at Bottom Fiber at Service Load

$$e_4 = -\frac{S_2}{A} + \frac{-\sigma_{ts} * S_2 + M_s}{P_e}$$

5. Eccentricity Controlled by Geometry of the Beam

$$e_{max} = c_2 - 2 = 6.76\text{ in}$$

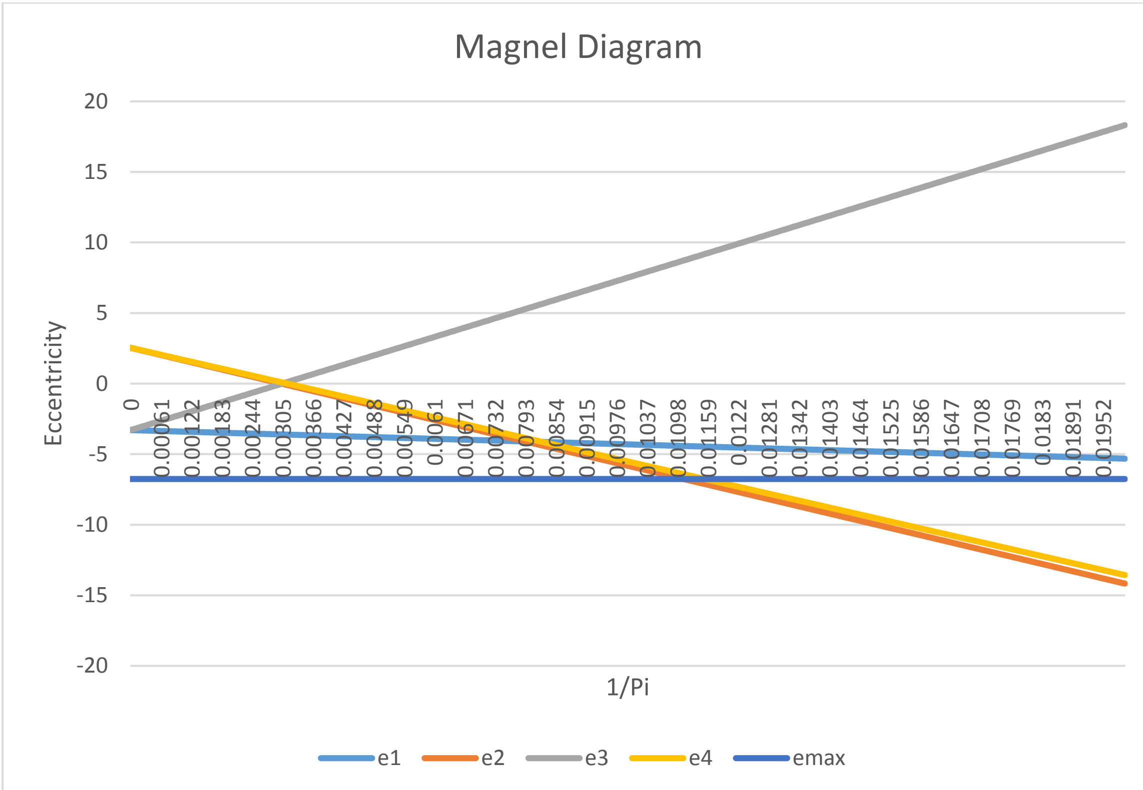

Figure B1: Mangel Diagram

Eccentricity used was e_{max}, e_1 was ignored by incorporating compression steel, see appendix B1.

B2. Ultimate Flexural Strength Design

1. Determine Moment Demand

Maximum Load $P_u = 36\ kips$

Maximum Moment $M_u = \frac{P_u * 84\ in}{2} + M_o = 1537.06\ k - in$

2. Determine Compression Zone Resultant in Flange

Depth of Prestressing $d_p = h - 2\ in = 13.5"$

Compressive Force from Concrete $\bar{C} = \frac{M_n}{d_p - \frac{a}{2}} = 120\ kips\ (found\ by\ iteration\ of\ a)$

Depth of Compression Zone Rectangle $a = \frac{\bar{C}}{(0.85 * f'c * b_f)} = 1.38\ in\ (found\ by\ iteration\ of\ \bar{C})$

3. Determine Location of Neutral Axis

Beta 1 Coefficient $\beta_1 = 0.65$ [ACI 318-11

Neutral Axis Depth

$c = \frac{a}{\beta_1} = 2.13\ in$

4. Determine Stress of Prestressing Steel

Ultimate Strain of Concrete $\epsilon_{cu} = 0.003$

Radius of Gyration Squared $r^2 = \frac{I}{A} = 22.2\ in^2$

Decompression Force $F_d = P_e \left[1 + \frac{E_p A_p}{E_c(A - A_p)}\left(1 + \frac{e^2}{r^2}\right)\right] = 26.8\ kips$

Prestrain of Prestressing Strand $Prestrain = \frac{F_d}{A_p E_p} = 0.002$

Ultimate Strain of Strand $\epsilon_{ps} = \epsilon_{cu}\frac{(d_p - c)}{c} + Prestrain = 0.018$

Stress of Prestressing Strand $f_{ps} = 262.5\ ksi\ \ *\ Using\ Actual\ f_{ps} vs.\ \epsilon_{ps}\ Relationship$

5. Determine if Mild Steel is Necessary

Tensile Force of Strand $T_p = A_p * f_{ps} = 120.5\ kips$

Tensile Force of Mild Steel $T_s = A_s * f_y$

Satisfy Equilibrium $T_p + T_s = \bar{C} \rightarrow T_p > \bar{C}$

$\therefore No\ mild\ Steel\ Needed, T_p = 120\ kips$

B3. Shear Design

1. Determine Shear Demand Along Beam

Support Reaction $P_{support} = P_u + \frac{w_o * L}{2} = 18.49\ kips$

Location along the Span of the Beam x

Shear Demand $V_u = P - w_o x$

2. Determine Shear Capacity of the Concrete

Moment Demand $M_u = Px - \frac{w_o * x^2}{2}$

Depth to Prestressing $d = d_p = 13.5\ in$

Concrete Shear Capacity $V_c = \left(0.6\sqrt{f_c'} + \frac{700(V_u * d_p)}{M_u}\right) b_w d$ [ACI 318-11 11.3.2]

Shear-Moment Ratio Upper Bound $\frac{V_u * d_p}{M_u} \leq 1$

3. Determine Minimum and Maximum Values of V_c

Strength Reduction Factor $\phi = 0.75$ [ACI 318-11 9.3.2.3]

Concrete Shear Capacity Upper Bound $V_{cmax} = 5\sqrt{f'c} b_w d = 26.8\ kips$ [ACI 318-11 11.3.2]

Concrete Shear Capacity Lower Bound $V_{cmin} = 2\sqrt{f'c} b_w d = 10.7\ kips$ [ACI 318-11 11.3.2]

4. Determine Shear Resistance Required From Stirrups

Steel Shear Capacity $V_s = \frac{V_u - \phi V_{cgovern}}{\phi}$ [ACI 318-11.1.1]

Steel Shear Capacity Upper Bound $V_{smax} = 13.8\ kips$

Steel Shear Capacity Check $V_{smax} = 8\sqrt{f'c} * b_w d = 44.2\ kips > 13.8\ kips \therefore Acceptable$

5. Determine Minimum Spacing of Stirrups

Stirrup Steel Area $A_v = 0.11\ sq.in$ (*Assumed 1 No. 3 Bar Vertical Leg)

Reinforcing Steel Yield Strength $A_{yt} = 60\ ksi$

Minimum Spacing $s_{min} = \frac{A_v * f_{yt} * d}{V_{smax}} = \mathbf{6.5\ in}$ [ACI 318-11 11.4.7.2]

6. Determine Where Maximum Spacing Could Be Used

Check $V_{smax} < 4\sqrt{f'c}b_w d = 22.1\ kips > 13.8\ kips \therefore Take\ Smallest\ of\ Three\ s_{max}\ Values$

First Maximum Stirrup Spacing Value $s_{max} = 0.75 * h = \mathbf{11.625\ in}$ [ACI 318-11.4.5.1]

Second Maximum Stirrup Spacing Value $s_{max} = 24\ in$ [ACI 318-11.4.5.1]

Third Maximum Stirrup Spacing Value $s_{max} = \frac{A_v * f_{yt}}{0.75 * \sqrt{f'c} * b_w} = 22.5\ in$ [ACI 318-11.4.6.3]

Maximum Stirrup Spacing Shear Force $V_s = \frac{A_v * f_{yt} * d}{11.625} = 7.66\ kips$

$*\ Large\ Enough\ until \sim 30"\ from\ Support$

7. Determine Where No Stirrups Could Be Used

No Stirrup Required Shear Force $\frac{\phi V_c}{2} = 3.9\ kips\ between\ point\ loads$

$*\ Large\ Enough\ Between\ Point\ Loads$

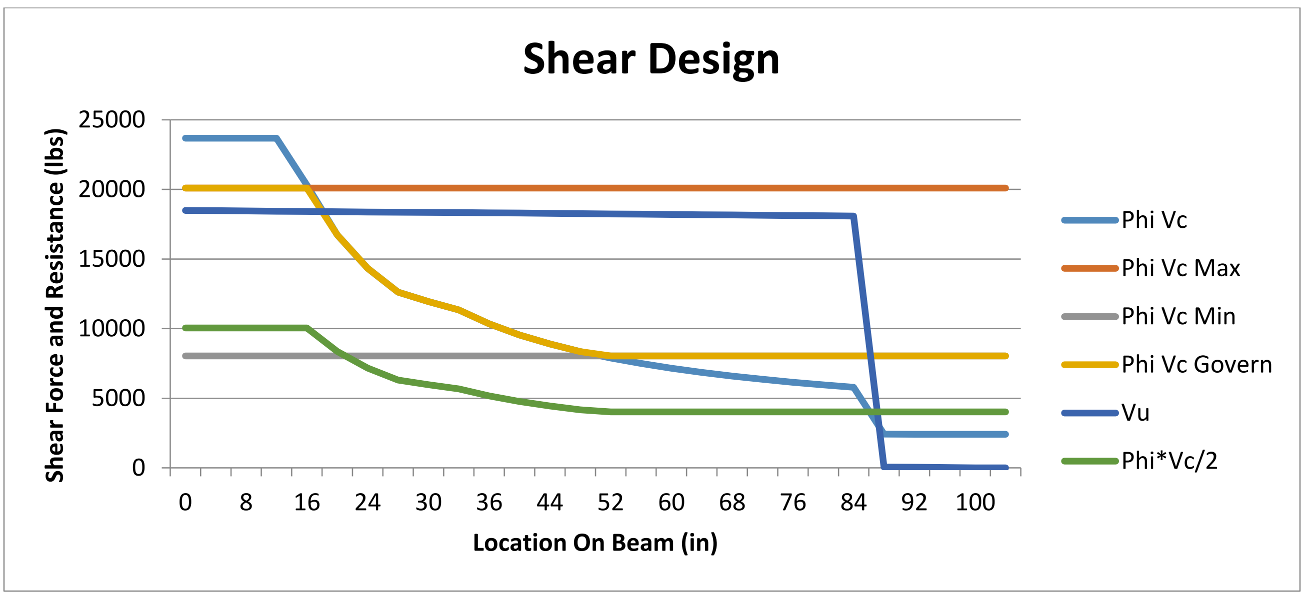

Figure B2: Shear Design Capacity vs. Demand

Final Spacing Decisions

1. Use $s = 11.625\ in\ until\ 28.75\ inches\ from\ Support$
2. Use $s = 6.5\ in\ until\ 80.75\ inches\ from\ Support$
3. Use no stirrups between point loads

B4. Design of Longitudinal Rebar to Resist Prestress Transfer

1. Determine Uncracked Transformed Area and Moment of Inertia
   Assume Flange Longitudinal Steel as 2 No.3 Bars

Area of Flange Longitudinal Steel $A_s = 0.22$

Modular Ratio $n = \frac{E_s}{E_c} = 3.83 \therefore Round\ up\ to\ 4$

Uncracked Transformed Area $A_{ut} = A + A_s(n-1) + A_{ps}(n-1) = 57.5\ in^2$

Area of Flange $A_f = h_f b_f = 18.0\ in^2$

Centroid of Flange from Top $y_f = \frac{h_f}{2} = 1.5\ in$

Area of Web $A_w = (h - h_f)b_w = 37.5\ in^2$

Centroid of Web from Top $y_w = \frac{h+h_f}{2} = 9.25\ in$

Centroid of Flange Longitudinal Steel $y_s = Cover + Stirrup\ Diamater + \frac{Bar\ Diameter}{2}$

$y_s = \frac{3}{8}in + \frac{3}{8}in + \frac{3}{16}in = 0.9375\ in$

Centroid of Uncracked Transformed Section $\bar{Y} = \frac{A_f y_f + A_w y_w + A_s y_s(n-1) + A_{ps} d_p(n-1)}{A + A_s + A_{ps}} = 6.83\ in$

Moment of Inertia of Flange about Centroid $I_f = \frac{b_f h_f^3}{12} = 13.5\ in^2$

Moment of Inertia of Web about Centroid $I_f = \frac{b_w(h-h_f)^3}{12} = 488.3\ in^2$

Moment of Inertia of Uncracked Transformed Section

$$I_{ut} = I_f + A_f(y_f - \bar{Y})^2 + I_w + A_w(y_w - \bar{Y})^2 + A_s(y_s - \bar{Y})^2(n-1) + A_{ps}(y_{ps} - \bar{Y})^2(n-1) = 1316.9\ in^4$$

2. Determine Location of Neutral Axis

Tensile Stress at Top Fiber at Prestressing $f_t = -\frac{P_i}{A} + \frac{P_i e_{max} c_1}{I_{ut}} + \frac{M_o c_1}{I_{ut}} = 1.37\ ksi$

Location of Neutral Axis Up from c_1 $f_t = 0 = -\frac{P_i}{A} + \frac{P_i \overline{y'} c_1}{I_{ut}} + \frac{M_o \overline{y'}}{I_{ut}} \therefore \overline{y'} = 3.5\ in$

Location of Neutral Axis From Top $\bar{y} = c_1 - \overline{y'} = 3.2\ in$

Assume Triangular Stress Distribution

Stress at Bottom of Flange $f_o = \frac{f_t}{\bar{y}}(\bar{y} - h_f) = 0.086\ ksi$

Tensile Force of Tensile Zone $T = \frac{b_w f_o(\bar{y} - h_f)}{2} + \frac{f_o + f_t}{2} * (h_f - \bar{y})b_f = 13.1\ kips$

Area of Steel Required $A_s = \frac{T}{f_y} = 0.21\ in^2 < 0.22\ in^2$

$\therefore 2\ No.\ 3\ Bars\ Acceptable$

APPENDIX C

PRETEST CALCULATIONS

C1. Cracking Strength Load

Experimental Concrete Tensile Strength $f_r = 1.506\ ksi$

Cracking Moment $M_{cr} = f_r S_2 + P_e\left(\frac{r^2}{c_2} + e_{max}\right) = 79.8\ kip - ft$

Associated Load $P_{cr} = \frac{M_{cr} - M_o}{L_t} = 11.1\ kips\ per\ point\ load$

C2. Ultimate Strength Load

Assumed Stress in the Prestressing Strands $f_{pf} = 0.9 f_{pu} = 243\ ksi$

Force in the Prestressing Strands $T_p = A_p f_{pf} = 115.37\ kips$

Force in the Mild Steel $T_s = 0$

Concrete Force $\overline{C} = T_p + T_s = 115.37\ kips$

Compression Block Depth $a = \frac{C}{0.85 f_c' b_f} = 1.17\ in$

Neutral Axis Depth $c = \frac{a}{\beta_1} = 1.80\ in$

Prestrain $\epsilon_{pre} = \frac{F_d}{A_p E_p} = 0.002$

Prestressing Strand Strain $\epsilon_{pf} = \frac{\epsilon_{cu}(d_p - c)}{c} + \epsilon_{pre} = 0.021 * Iterated\ on\ in\ spreadsheet$

Final Prestressing Strand Strain $\epsilon_{pf} = 0.00196$

Final Stress in Prestressing Strands $f_{pf} = 263.788\ ksi$

Nominal Moment Strength of Strands $M_n = T_p\left(d_p - \frac{a}{2}\right) + T_s\left(d - \frac{a}{2}\right) = 130.616\ kip - ft$

Load Associated with Nominal Moment $P_n = \frac{M_n - M_o}{L_t} = 18.361\ kips\ per\ point\ load$

C3. Deflection

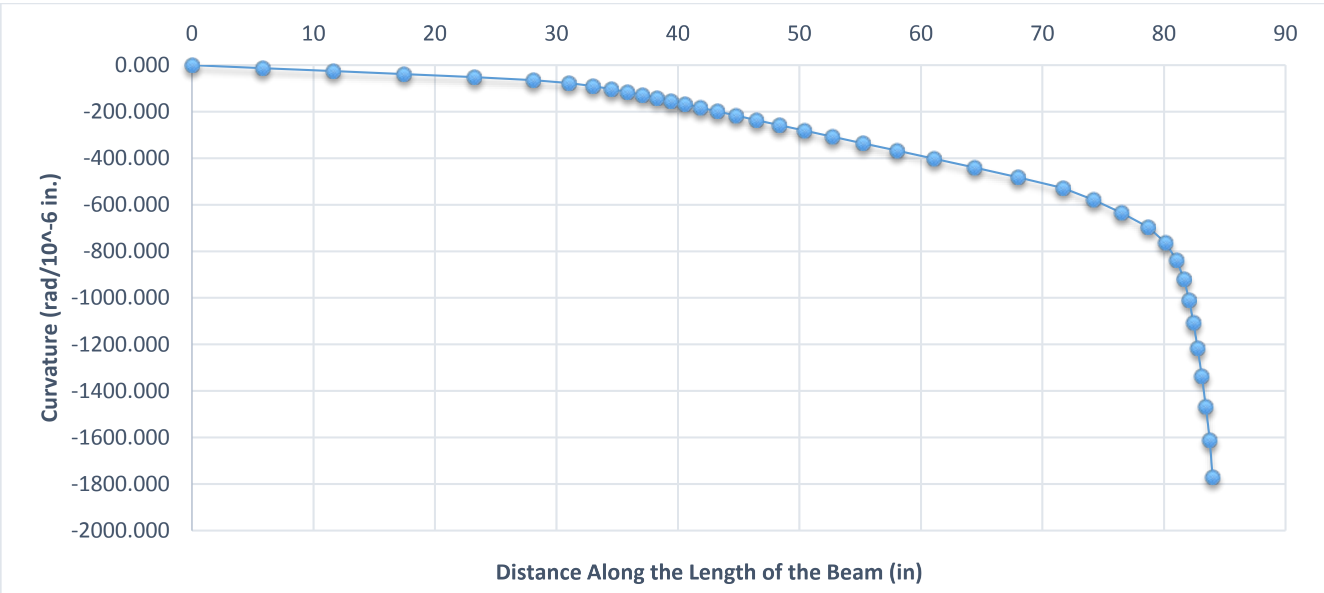

Figure C1: Curvature along the Length of the Beam

Table C1: Deflection Calculation				
Distance (in.)	Curvature (rad)	Area (in^2)	Centroid to Center of beam	Delta
0.00	0.0000000	0.0000375	98.12	0.00368
5.81	0.0000129	0.0001125	92.95	0.01046
11.63	0.0000258	0.0001876	87.27	0.01637
17.44	0.0000387	0.0002626	81.51	0.02141
23.26	0.0000516	0.0002805	76.24	0.02138
28.09	0.0000645	0.0002084	72.40	0.01509
31.03	0.0000774	0.0001645	69.97	0.01151
32.99	0.0000903	0.0001482	68.23	0.01011
34.52	0.0001032	0.0001460	66.80	0.00975
35.85	0.0001161	0.0001511	65.52	0.00990
37.08	0.0001290	0.0001592	64.32	0.01024
38.26	0.0001419	0.0001695	63.16	0.01071
39.40	0.0001548	0.0001881	62.01	0.01167
40.57	0.0001682	0.0002234	60.79	0.01358
41.84	0.0001829	0.0002673	59.45	0.01589
43.24	0.0001990	0.0003184	57.99	0.01846
44.77	0.0002168	0.0003836	56.37	0.02162
46.46	0.0002363	0.0004613	54.59	0.02518
48.33	0.0002578	0.0005594	52.62	0.02943
50.40	0.0002814	0.0006735	50.43	0.03397
52.69	0.0003075	0.0008140	48.02	0.03909
55.22	0.0003361	0.0009820	45.36	0.04454
58.01	0.0003675	0.0011774	42.43	0.04996
61.07	0.0004021	0.0014065	39.23	0.05518
64.41	0.0004402	0.0016460	35.78	0.05889
67.98	0.0004821	0.0018648	32.14	0.05994
71.67	0.0005282	0.0013961	29.05	0.04055
74.20	0.0005789	0.0013987	26.63	0.03725
76.50	0.0006346	0.0014509	24.39	0.03539
78.68	0.0006959	0.0010541	22.58	0.02381
80.13	0.0007634	0.0007279	21.41	0.01558
81.04	0.0008376	0.0005270	20.66	0.01089
81.64	0.0009192	0.0004216	20.14	0.00849
82.07	0.0010090	0.0003795	19.74	0.00749
82.43	0.0011077	0.0003966	19.39	0.00769
82.77	0.0012164	0.0004364	19.05	0.00831
83.12	0.0013359	0.0004640	18.72	0.00868
83.45	0.0014673	0.0005038	18.39	0.00926
83.77	0.0016119	0.0003834	18.11	0.00694
84.00	0.0017710	0.0318771	9.00	0.28689
102.00	0.0017710			
	Sum:	0.054410583	**Sum:**	1.1352822
	Total Deflection:	4.414597215 in		
	Camber:	0.937 in		
	Deflection of Beam:	5.351597215 in		

C4. Percentage Error of Predictions

Predicted Maximum Applied Load	$P_{np} = 2P_n = 36.7\ kips$
Actual Maximum Applied Load	$P_{na} = 38.5\ kips$
Maximum Applied Load Error	$e_{P_n} = \frac{\lvert P_{np} - P_{na}\rvert}{P_{np}} = 0.049$
Predicted Cracking Load	$P_{crp} = 2P_{cr} = 22.2\ kips$
Actual Cracking Load	$P_{cra} = 22.5\ kips$
Cracking Load Error	$e_{P_{cr}} = \frac{\lvert P_{crp} - P_{cra}\rvert}{P_{crp}} = 0.014$
Predicted Maximum Deflection	$\Delta_{maxp} = 5.35\ in$
Actual Maximum Deflection	$\Delta_{maxa} = 5.44\ in$
Maximum Deflection Error	$e_{\Delta} = \frac{\lvert \Delta_{maxp} - \Delta_{maxa}\rvert}{\Delta_{maxp}} = 0.017$
Total Prediction Error	$e_t = e_{P_n} + e_{P_{cr}} + e_{\Delta} = 0.080$

APPENDIX D

ADDITIONAL PHOTOGRAPHS

Figure D.1a No Load

Figure D.1b Uncracked Under Load

Figure D.1c Initial Cracking

Figure D.1d Further Cracking

Figure D.1e Extensive Cracking

Figure D.1f Ultimate Failure

Beam Loading Sequence

Figure D.1g

Figure D.1f

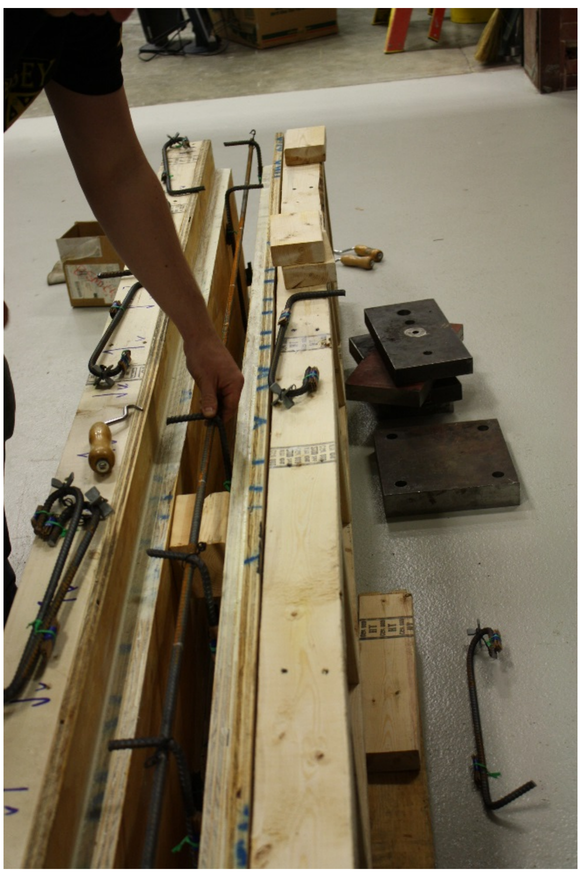

Figure D.1h

Figure D.1i

Figure D.1j

Formwork Construction Pictures

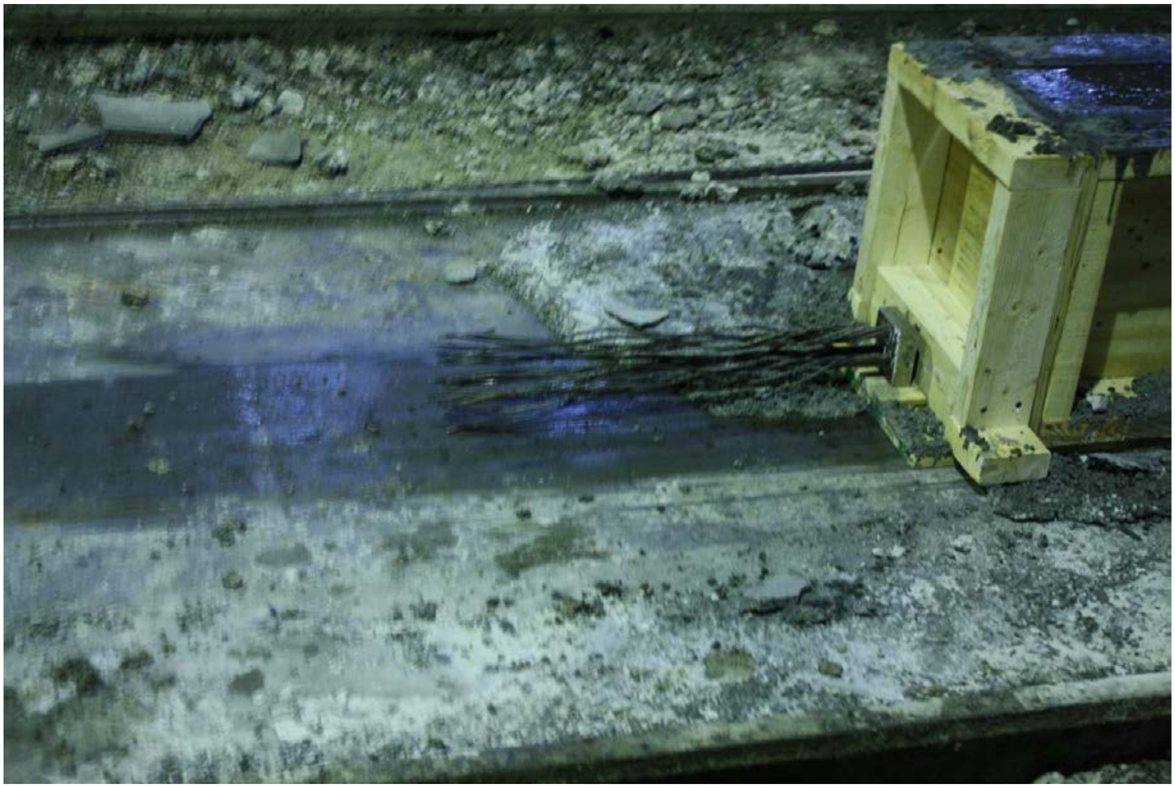

Figure D.1k

Prestressing Strand after Being Cut