PCI Big Beam Report 2016

University of Notre Dame

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Summary of Results

PCI BIG BEAM COMPETITION 2016

April 4, 2016
Date

University of Notre Dame
Student Team (school name)

Team Number

January 29, 2016
Date of Casting

Basic information

1. Age of beam at testing (days) 32

2. Compressive cylinder tests*
   - Number tested: 4
   - Size of cylinders: 3 in. x 6 in.
   - Average: 14,950 psi

3. Unit weight of concrete (pcf) 150
   - Slump (in.): 8+
   - Air content (%): 0.9
   - Tensile strength (psi): 1,273
   - Circle one: Split cylinder MOR beam

4. Pretest Calculations
   - a. Applied point load at midspan to cause cracking (kip) 17.50
   - b. Maximum applied point load at midspan (kip) 31.20
   - c. Maximum anticipated deflection due to applied load only (in.) 2.31

Pretest calculations MUST be completed before testing.

*International entries may substitute the appropriate compressive strength test for their country.

Test summary forms must be included with the final report, due June 17, 2016

Judging Criteria

Teams MUST fill in these values.

a. Actual maximum applied load (kip) 32.73

b. Measured cracking load (kip)* 19.41

c. Cost (dollars) 114.08

d. Weight (lb) 863.2

e. Largest measured deflection (in.) 2.56

f. Most accurate calculations

(a) Absolute value of (maximum applied load – calculated applied load) / calculated applied load 0.049

(b) Absolute value of (maximum measured deflection – calculated deflection) / calculated deflection 0.109

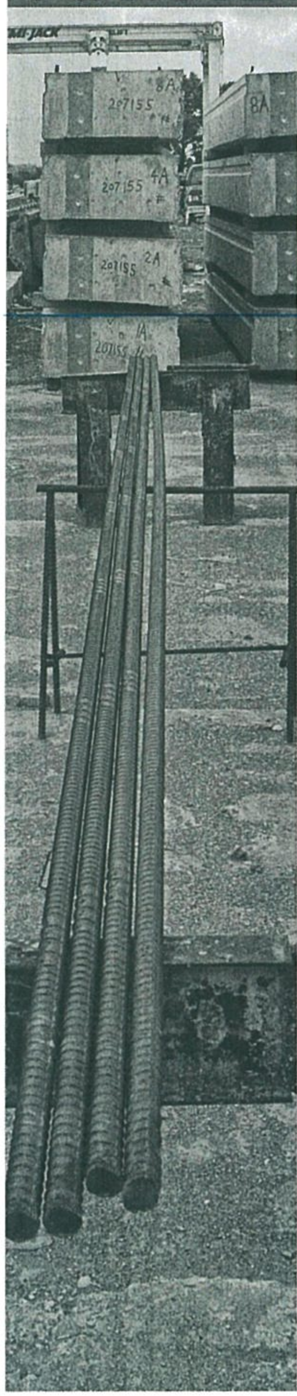
(c) Absolute value of (measured cracking load – calculated cracking load) / calculated cracking load 0.111

Total of three absolute values (a + b + c) = 0.269

*Measured cracking load is found from the "bend-over" point in the load/deflection curve. Provide load/deflection curve in report.

Certification

PCI BIG BEAM COMPETITION 2016



CERTIFICATION

StresCore, Inc.

As a representative of (name of Producer Member or sponsoring organization)

University of Notre Dame

Sponsoring (name of school and team number)

I certify that:

- The big beam submitted by this team was fabricated and tested within the contest period.
- The calculations of predicted cracking load, maximum load, and deflection were done prior to testing of the beam.
- The students were chiefly responsible for the design.
- The students participated in the fabrication to the extent that was prudent and safe.
- The submitted test results are, to the best of my knowledge, correct, and the video submitted is of the actual test.

Certified by:

Signature

Jason Reihl

Name (please print)

5/24/16

Date

THIS CERTIFICATION MUST BE PART OF THE FINAL REPORT

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Drawings and Geometric Design

Three cross-sectional shapes were considered for the design of the beam: (1) rectangular, (2) I-beam, and (3) T-beam. Based on discussions with the faculty advisor and also knowledge on reinforced concrete beams under ultimate loads, the T-beam cross section was determined to be the most effective and efficient cross section for the competition. The beam dimensions, such as total depth, web thickness, flange thickness, and flange width, were iterated upon in order to obtain optimum dimensions to carry the prescribed loads and cost the least to fabricate. The costs of the formwork, steel, and concrete were taken into account when determining the beam dimensions. In addition, the weight of the beam was considered, and the cross section was chosen to minimize the concrete required. Figure 1 below shows the final dimensions of the beam which has a uniform cross section along its length. Figure 2 shows the cross section of the constructed formwork. Figure 3 depicts the spacing of the shear reinforcement along the length of the beam.

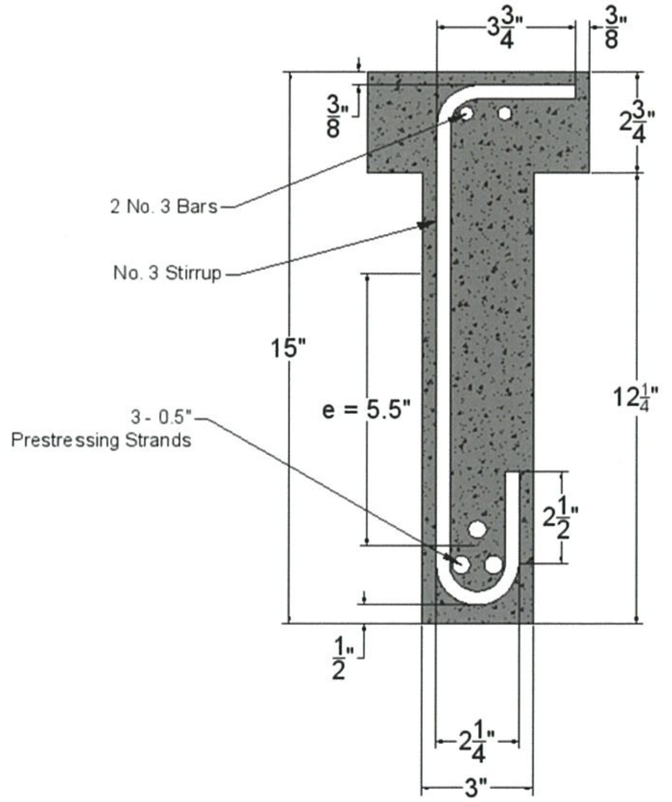

Figure 1: Final Beam Dimensions

Figure 2: Beam Formwork

Pre-tensioning, rather than post-tensioning, was selected as the method for prestressing the beam. Straight prestressing strands, as opposed to harped strands, were chosen because the pre-tensioning bed was not configured to harp the strands. The beam span from support to support was 15 feet and extended an additional 8 inches beyond each support for stability purposes. This resulted in a total length of 16 feet 4 inches which satisfied the maximum 17 feet allowed in the competition rules. For further information about the beam dimensions, refer to Appendix A.

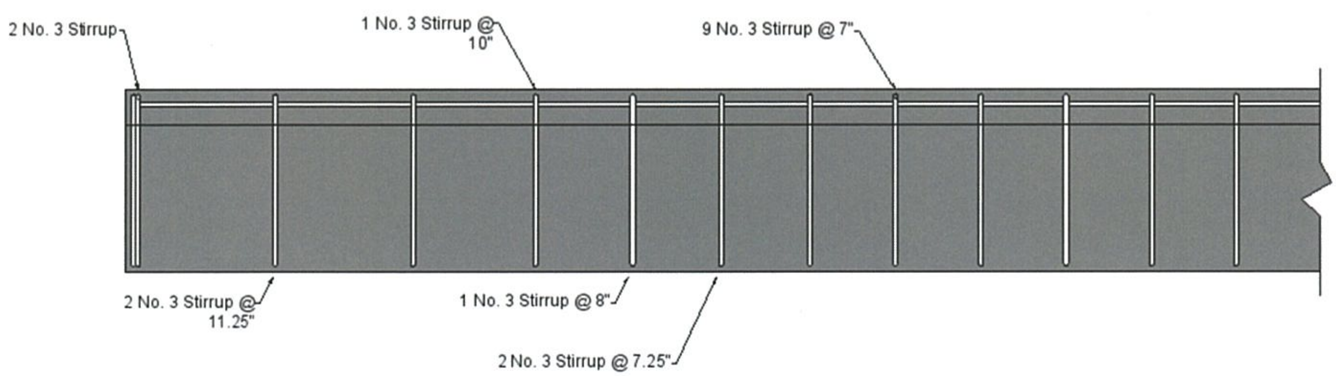

Figure 3: Spacing of shear reinforcement along beam length

Concrete Mix Design

The goal of the concrete mix design was to achieve workable concrete with a compressive strength of about 17,000 psi to minimize the size, and therefore the volume, weight, and formwork of the beam. Five different concrete mix designs were batched, with their constituent components and properties listed in Table 1. The first four mixes were similar, with only a few variables changed at a time.

INDOT #11 limestone with 3/8 inch nominal maximum aggregate size was used as coarse aggregate. The amount of high range water reducer was kept constant throughout the different mixes. Mix 1 suffered from segregation issues and was discarded. Mix 2 focused on decreasing the limestone content and increasing the amount of water to increase the workability of the mix. Mix 3 had decreased water-to-cementitious materials (w/c) ratio by reducing the volume of water in an effort to further increase the concrete strength. Mix 4 was designed to reduce the total volume of coarse and fine aggregates and increase the cementitious material content. Finally, Mix 5 was designed to include polyvinyl alcohol (PVA) fibers in order to increase the tensile strength of the concrete. The use fibers resulted in a significant reduction in compressive strength; and therefore this mix was not pursued further.

Table 1: Laboratory Trial Concrete Mix Designs

Material	Mix 1	Mix 2	Mix 3	Mix 4	Mix 5
Portland Cement Type I	950	950	950	950	864
INDOT #11 Limestone	3680	1830	1830	1800	1821
INDOT #23 Sand	1300	1300	1300	1000	1155
Silica Fume	90	90	90	150	73
Fly Ash	50	50	50	0	0.0
Water	255	280	250	255	219
Nycon PVA 8mm fibers	0	0	0	0	1.15
High Range Water Reducer	36	36	36	36	36
Water/Cement	0.23	0.26	0.23	0.23	0.23

Nine 3 x 6 inch concrete cylinders were made using each concrete mix to determine its average compressive strength. A standard slump test following ASTM C143/143M was performed to determine the workability of each mix. All batching and testing of trial mixes were done in the University of Notre Dame Structural Systems Laboratory and adhered to ASTM C39/C39M and ASTM C143/C143M guidelines. Three cylinders were tested at each of the 3-day, 7-day, and 28-day ages. The average 28-day compressive strengths of the concrete mixes can be found in Table 2.

Mix 3 was selected as the final mix to be used in the casting of the beam specimen due to its large compressive strength of 17,425 psi and high workability. This compressive strength exceeded that of the target design.

The concrete used for the casting of the beam was prepared and delivered by Ozinga Ready Mix Concrete in South Bend, Indiana. In addition to the beam, sixteen 3 x 6 inch cylinders and 3 modulus of rupture (MOR) beams were cast for material testing. These were stored and cured under the same conditions as the beam. Four cylinders were tested at each of the 3-day, 7-day, 28-day, and beam-testing-day (32 day) ages. The MOR beams were tested at the 28-day age. The results from these tests and some additional information of the concrete can be found in Table 3. The cylinders were weighed and their actual dimensions were measured to obtain an average unit weight of 153 pcf.

Table 2: Average 28-Day Strength

Mix No.	Compression Strength (psi)
1	N/A
2	14,530
3 (lab cast)	17,430
4	12,540
5	11,580
3 (Ozinga)	0

Table 3: Concrete Properties of Cast Beam (Ozinga)

Slump (in.)	>8
Tensile Strength, MOR (psi)	1,273
Air Content	0.9%
Unit Weight (pcf)	153
7 Day Compressive Strength (psi)	12,870
14 Day Compressive Strength (psi)	13,080
28 Day Compressive Strength (psi)	14,950
Beam Test Day Compressive Strength (psi)	14,720

The compressive strength of the samples taken from the Ozinga ready-mix concrete delivery was weaker than that of the laboratory mix at Notre Dame. After closely observing the samples taken from the ready-mix concrete, there was segregation between the coarse aggregate and the cementitious paste that may have caused the reduction in the compressive strength. Segregation was also observed in the upper layer of the beam flange as well, as seen in Figure 4 below. The 28-day compressive strength of the ready-mix concrete used to cast the beam was used to conduct the pre-test predictions of the beam cracking load, ultimate load, and maximum deflection.

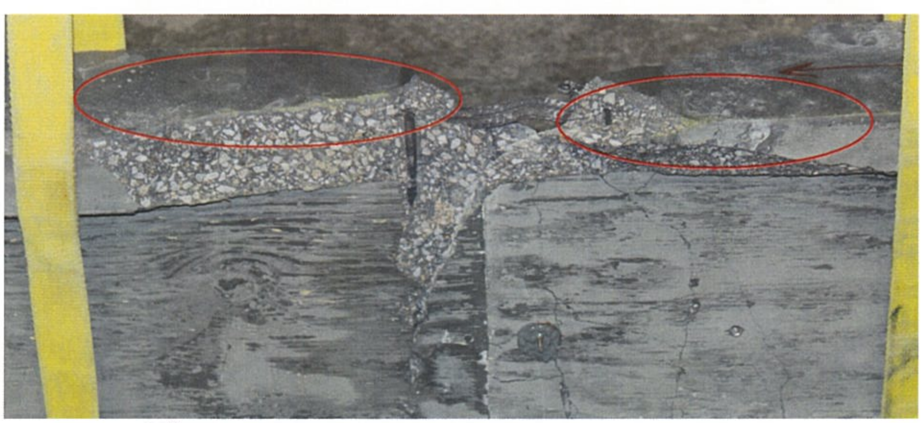

Figure 4: Segregation of Concrete in Beam Flange

Structural Design

Allowable Stress Design

The competition rules require the beam to carry a superimposed service load of 18.75 kips (in addition to self-weight) applied at the midspan without cracking. Because of expected variabilities in the concrete tension strength, the beam was conservatively designed not to crack under a service load of 20 kips. This load, combined with the self-weight of the beam, resulted in a total midspan service moment, M_s, of 918.7 kip-inches.

The allowable stresses prescribed by ACI 318-14 result in prestress force, P, and eccentricity, e, requirements at the prestress transfer and service load stages. The allowable eccentricities for the prestress transfer stage are e_1 corresponding to the allowable tension stress at the top of the beam and e_2 corresponding to the allowable compression stress at the bottom of the beam. In addition, the allowable eccentricities for the service load stage are e_3 for the compression stress at the top of the beam and e_4 for the tension stress (to prevent cracking) at the bottom of the beam. ACI 318-14 indicates that e_1 may be violated if deformed steel reinforcement (Grade 60) is included in the top flange to resist the tension force developing in the flange at prestress transfer. The top flange reinforcement design is described later in this report.

The detailed allowable stress calculations for the beam can be found in Appendix B. The prestress force, P was governed by the allowable tensile stress (to prevent cracking under service loads) at the bottom of the beam with the prestressing tendon placed at the maximum eccentricity, e_{max} possible for the given cross-section (i.e., e_4=e_{max}). The allowable stress in tension was taken from ACI 318-14 Table 24.5.2.1 resulting in a tension strength of 978 psi for the concrete. Note that the as built tension strength of the concrete was considerably higher than the result from the ACI 381-14 Table 24.5.2.1 of $7.5\sqrt{f_c'}$ (where, f_c' is the concrete compression strength) because of the use of high-strength concrete (the ACI equation generally underestimates the tension strength of high-strength concrete). The allowable stresses for concrete in compression did not govern the design.

With the strands placed inside the hooks of the shear reinforcement, the maximum eccentricity, $e_{max,}$ was determined by placing the tendon centroid 1.5 inches from the bottom of the cross section as shown in Figure 1. Assuming an initial prestress of 70% of the ultimate strength (270 ksi) of the strands and a prestress loss reduction factor of 0.925 at service loads, the final allowable stress design resulted in three 0.5-inch diameter strands. This is depicted in the Magnel Diagram of the allowable stress design in Figure B1 of Appendix B which was used as a graphical design aid.

The choice of the 0.5-inch diameter strands was based on the availability of this strand size at the precast production plant where the beam was cast. The three strands were low-relaxation uncoated steel prestressing strands and were placed in a triangular arrangement as shown in Figure 1 to fit within the hooks of the shear reinforcement and also match the position of existing holes on the abutment bulk-heads of the pre-tensioning bed. The resulting initial prestress force at transfer, P_i, was 86.75 kips.

Ultimate Flexural Strength Design

The competition rules state that the ultimate strength of the beam should be adequate to carry a superimposed ultimate load of between 30 and 37 kips applied at the midspan, in addition to self-weight. To satisfy these limits, the beam was designed to have an ultimate strength excluding self-weight of 33.5 kips using Chapters 9, 20, and 22 of ACI 318-14. The design calculations were done to determine if any deformed (Grade 60) steel reinforcement was needed at the bottom of the beam to provide for this ultimate strength in addition to the three prestressing strands placed based on the allowable strength design. It was found that the three 0.5-inch diameter prestressing stands were adequate and so the final beam design did not include any Grade 60 steel bars at the bottom of the beam. The detailed ultimate strength design process can be found in Appendix B.

Shear Design

Shear design for the beam followed Chapter 22 of ACI 318-14. Because of the small web width of the beam, a single-leg stirrup was selected. The stirrups were made from No. 3 bars, with a 90-degree hook at the top (inside the flange) and a 180-degree hook at the bottom (web), as seen in Figure 1. The stirrups were placed in alternating directions, with the top and bottom hooks facing one side of the beam and then the other side of the beam for adjacent stirrups, to minimize out-of-plane asymmetry for the beam. A clear cover of ½ inch was used at the bottom and a clear cover of ⅜ inch was at the sides of the web and the top of the flange.

The detailed calculations for the beam shear design can be found in Appendix B. The ultimate shear force, V_u, was calculated by summing the shear caused by the ultimate superimposed design load of 33.5 kips applied at the midspan and the self-weight of the beam. The shear strength of the concrete, V_c, as well as its maximum and minimum limits were calculated in accordance with ACI 318-14. Because of the use of high-strength concrete, the concrete contribution to the shear resistance of the beam was large. A capacity reduction factor of ϕ=0.75 was used to determine the shear strength, V_s , The maximum stirrup spacing of 11.25 inches allowed by ACI 318 governed the spacing of the stirrups near the supports. Away from the supports, the increased bending moment resulted in a reduction in V_c and therefore an increase in V_s. The smallest required stirrup spacing was 7 inches and was used near the midspan of the beam, as can be seen in Figure 3.

Longitudinal Mild Steel Design to Resist Flange Stresses at Transfer

As discussed in the Allowable Stress Design section, the tension stress at the top of the beam under prestress transfer exceeded the allowable stress from ACI 318. As required by ACI 318 for this condition, the beam was reinforced with longitudinal rebar in the flange to resist the tensile stresses from prestress transfer. These stresses were assumed to have a linear distribution from the top of the flange to the neutral axis. A tensile stress resultant was then found by integrating the assumed triangular stress distribution over the region of the cross-section in tension. The concrete was assumed to contribute no tensile strength, so the required area of the reinforcement was found by dividing the tension resultant by the yield strength of the steel. The required steel area found was 0.24 in^2 which is less than 10% greater than 2 No. 3 bars. Since the design was conservative in assuming the concrete had no tension strength, it was determined that two Grade 60 No. 3 bars (ASTM A615) would be sufficient for the final design. The detailed design process and calculations can be followed in Appendix B.

Beam Fabrication

Formwork Design and Construction


Figure 5: Formwork Construction

While constructing the formwork, all safety protocols established by the University of Notre Dame and the Structural Systems Laboratory were followed. This included wearing safety goggles, long pants, and closed toed shoes when working as seen in Figure 5, as well as being trained to use any machinery and power equipment. The construction team worked with the design team in a design-build interaction to determine the formwork details. This helped optimize the cost efficiency of the formwork.

The shape and dimensions of the formwork which were represented in AutoCAD, are shown in Figures 6 and 7. Using the AutoCAD drawings, the necessary quantities of lumber and supplies for the formwork were finalized. Standard 2-inch by 4-inch lumber and standard ¾ inch thick plywood were used to save on the cost of supplies. The sides and bottom of the formwork were cut from 4 feet by 8 feet plywood sheets and reinforced with 2-inch by 4-inch lumber at a spacing of 11 inches.

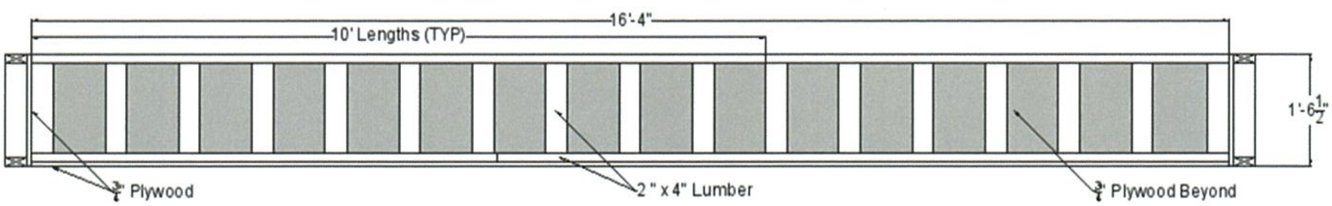

Figure 6: Longitudinal Formwork Layout

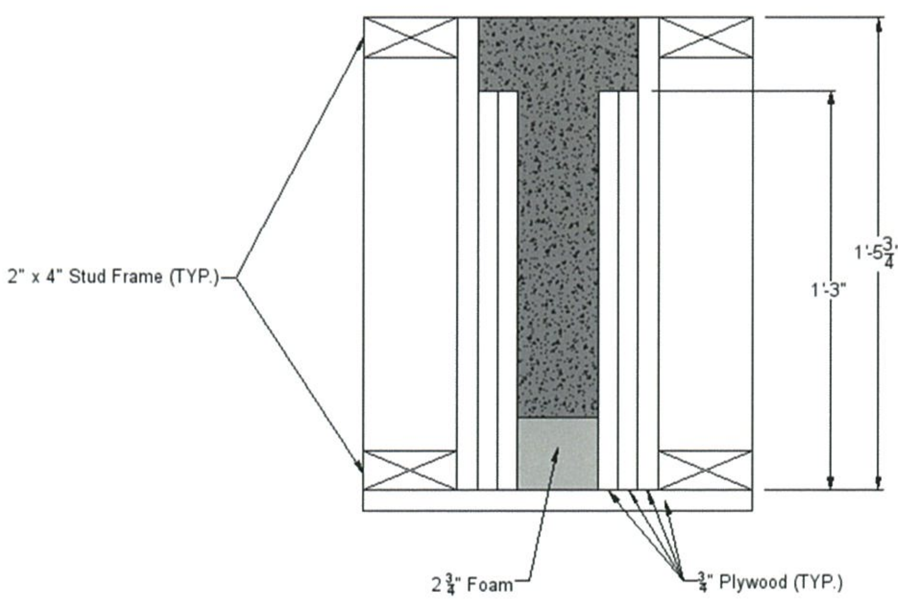

Figure 7: Formwork Cross-Section

Rebar Cage Assembly and Placement

The rebar cage was assembled inside the formwork using the design reinforcement with two longitudinal No. 3 bars in the flange and single-leg vertical stirrups distributed along the length, as shown in Figure 3. The stirrups were placed with the hooks in an alternating pattern and were wire-tied to the two longitudinal bars in the flange. Lab-made chairs were use on the bottom and sides of the formwork to ensure the desired placement of the stirrups with adequate cover.

On the day of casting the beam, three 0.5-inch diameter steel prestressing strands were passed through the bottom hooks of the stirrups. The final positioning and spacing of the stirrups were then re-checked/adjusted to ensure that the stirrups were not shifted from their intended positions and alignment prior to casting.

Beam Casting and Prestressing

The beam was cast on Friday, January 29, 2016 on a heated prestressing bed at StresCore, Inc. in South Bend, Indiana. StresCore Inc. is a PCI-certified plant that specializes in hollow core panels. All safety protocols were followed, including wearing safety goggles, hard hats, and hard-toe closed-shoes during the plant operations. The team worked together to pass the three strands through the formwork and adjust the rebar cage as seen in Figure 8. The strands were then pre-tensioned by Stresscore staff using bulkheads at each end of the bed. Figure 9 shows the concrete delivery during casting day.

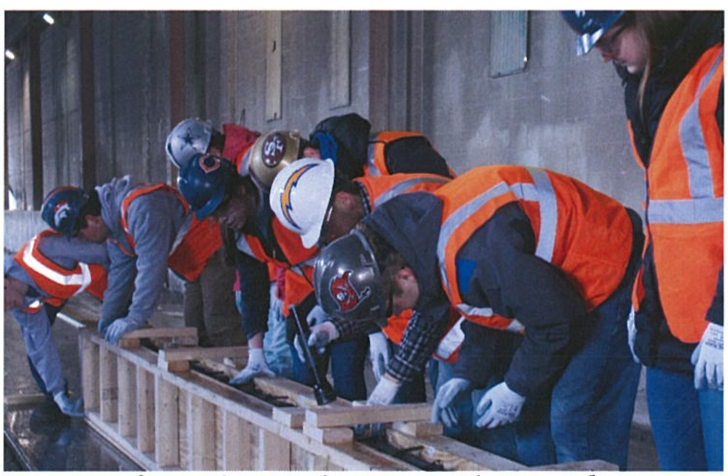

Figure 8: Inserting Prestressing Strands

Figure 9: Concrete Delivery

The concrete was poured directly from the ready mix concrete truck into the formwork. Vibration rods were used to eliminate air pockets, taking care to not over-vibrate the concrete. Trowels were then used to finish the top surface of the beam as seen in figure 10, and the beam was left on the heated bed and covered under tarp so that the concrete could set. A slump test of the concrete was performed during casting day as observed in Figure 11. On Monday, February 1, 2015, the team viewed the releasing of the pre-tensioned strands by the staff at StresCore. The strands were released from the live end one at a time in order to successfully transfer the prestress force into the concrete. The beam was then transported to the Structural Systems Laboratory at Notre Dame and stored indoors until the testing day.

Figure 10: Concrete Finishing

Figure 11: Slump Testing on Casting Day

Pretest Predictions

Cracking Moment

The pre-test prediction for the cracking moment of the beam was made by reversing the process that was used in design. The measured concrete properties on test-day and the as-built dimensions of the beam replaced the design values. The as built dimensions of the beam were identical to the design values. The as built eccentricity of 5.50 inches, however, differed from the design value of 6.70 inches. This happened because of the inaccuracy of drilling the holes that would pass the prestressing strands through the cross section of the beam. This contributed to the error in the ultimate strength of the beam. The properties of the concrete used to cast the beam were measured using compression cylinder and modulus of rupture (MOR) beam tests, and were used to finalize the pre-test calculations prior to the testing of the beam. By averaging the MOR test results, the concrete was found to have a tension strength of 1,273 psi which was greater than the aforementioned value of 978 psi taken from ACI 318-14.

The predicted cracking moment using the as-built dimensions and concrete property measurements was 67.2 k-ft, and the associated applied superimposed load (not including beam self-weight) was found to be 17.5 kips. The discrepancy between the 20 kip design load (see section on Allowable Stress Design) and the 17.5 kip predicted load for cracking occurred because of the smaller as-built eccentricity of the prestressing strands than the design eccentricity. Detailed pre-test calculations for the cracking moment can be seen in Appendix C.

Ultimate Strength

Similarly, the ultimate flexural strength was predicted by reversing the design procedure while using the as-built dimensions and average concrete material properties of the beam (Appendix C). Assuming that the two No. 3 bars in the flange yielded in compression, the stresses and strains in the beam were iterated upon until the stress in the prestressing steel, f_{pf}, agreed between the calculations and the assumed stress-strain behavior of the strands. The yielding of the No. 3 bars was validated from the strain diagram of the beam. From there, the ultimate flexural strength, M_n, was determined through flexural equilibrium as M_n = 118.6 k-ft, with the associated superimposed load (not including self-weight) being 31.2 kips. Similar to the cracking load, the smaller as-built eccentricity of the strands resulted in a smaller predicted ultimate load than the design strength of 33.5 kips, but the actual strength of the beam still satisfied the competition requirement. The compression strength of the as-built concrete 14,950 psi on test-day was somewhat smaller than the design value 17,000 psi, but this difference had a relatively small effect on the ultimate strength of the beam.

Maximum Deflection at Ultimate Strength

The maximum deflection of the beam at midspan was estimated using moment-area theorems which related the moment diagram along the length of the beam to the moment versus curvature relationship of the prestressed cross-section. The maximum moment was determined from the pre-test calculations for the ultimate strength, and the nonlinear moment-curvature relationship of the cross-section was generated based on the as-built dimensions and material properties using the structural analysis program, Response 2000 (Bentz and Collins 2000). The moment diagram was discretized along the length of the beam, and the nonlinear moment-curvature relationship from Response 2000 was used to find the corresponding curvature at each discretization point. The resulting curvature diagram along the beam length was assumed to be linear between the discretization points. Since the curvature gradient was greater near the beam midspan, a greater number of discretization points spaced closer together was used in this region. The moment-area theorems were then used to determine a predicted a midspan deflection of 1.182 inches. The initial camber of 1.125 inches measured for the beam was then added to the calculated deflection, resulting in a total deflection of 2.307 inches from the initial position of the beam. The detailed prediction calculations can be seen in Appendix C.

Beam Testing

Testing of the beam occurred on March 1, 2016, at an age of 32 days after casting. The beam was simply supported with steel wedge supports placed 15 feet apart, conforming to the required competition clear span. A hydraulic jack with an integrated load cell applied the load at the midspan of the beam. The load cell in the hydraulic jack measured the applied superimposed load and two linear potentiometers on each side of the beam below the flange were used to measure the mid-span deflection. The testing setup can be observed in Figure 12 below.

Cracking Load

The beam started with an upward camber of about 1.125 inches and began deflecting linearly as expected. The bend-over point on the measured load versus deflection curve (i.e., where the load-deflection curve deviates from the black line segment as seen in Figures 14 and 15) showed a superimposed load of 19.41 kips at cracking. Thus, the minimum required competition service load of 18.75 kips was met.

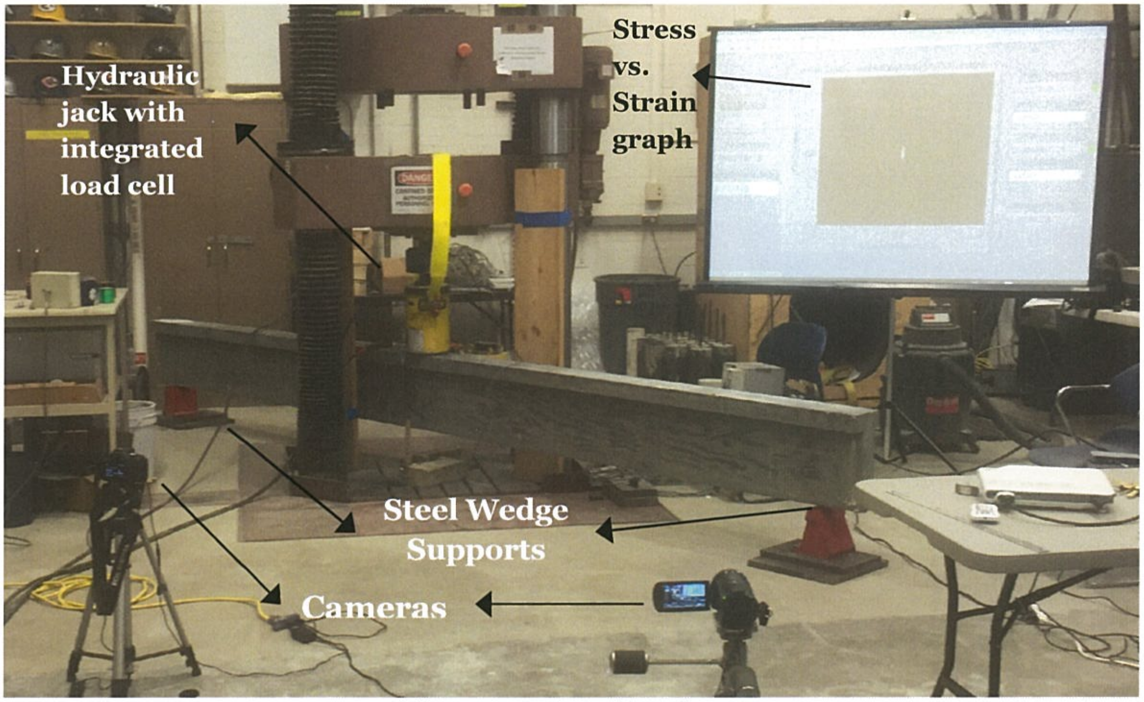

Figure 12: Beam Test Setup

Peak Load and Ultimate Failure

As the load on the beam was increased, flexural cracks began to appear at the bottom of the web below and on both sides of the midspan (i.e., point of load application). This was expected as the maximum moment was at the midspan of the beam. As the load was increased, the cracks propagated up the web and widened, as seen below in Figure 13. At ultimate failure, the concrete in the flange of the beam began to crush after the prestressing steel had yielded, resulting in a ductile flexural failure. This was in accordance with the design of the beam, as ductile flexural failure is desirable. After analyzing the data, the ultimate failure of the beam occurred at a maximum superimposed load of 32.73 kips at the midspan of the beam. This met the ultimate load range of 30 to 37 kips from the competition requirements.

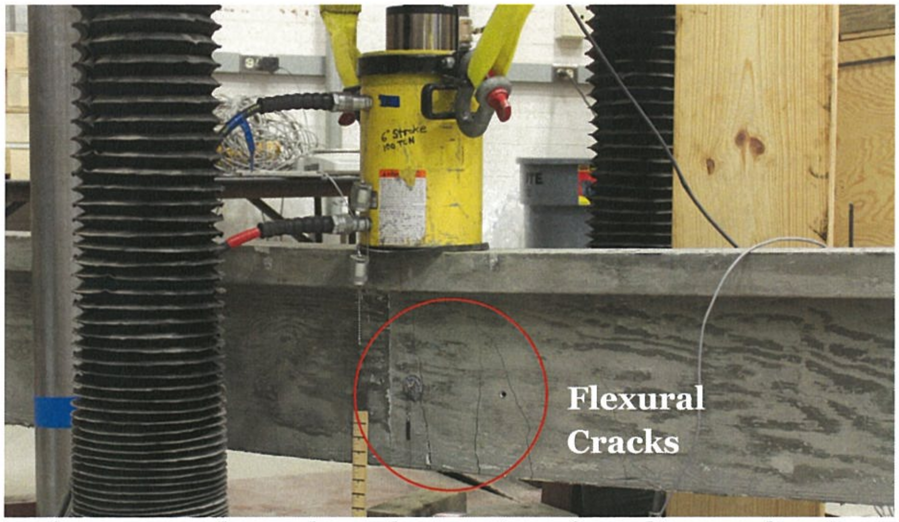

Figure 13: Flexural Cracks at Point of Loading Application

Maximum Deflection

The beam experienced a deflection of 0.343 inches under the service load of 18.75 kips. This deflection is equivalent to 0.19% of the span length which is under the ASCE 7-10 total load deflection limit (L/240) of 0.75 inches. The total deflection of the beam at ultimate failure, coinciding with the maximum load, was 2.564 inches which is slightly greater than the predicted value of 2.307 inches.

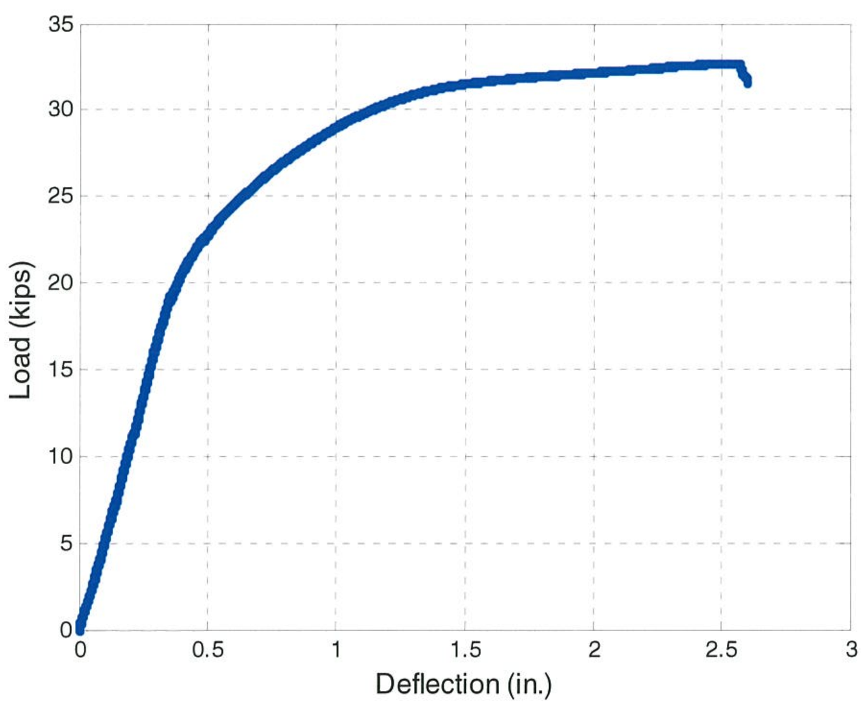

Figure 14: Measured Superimposed Load versus Deflection Curve

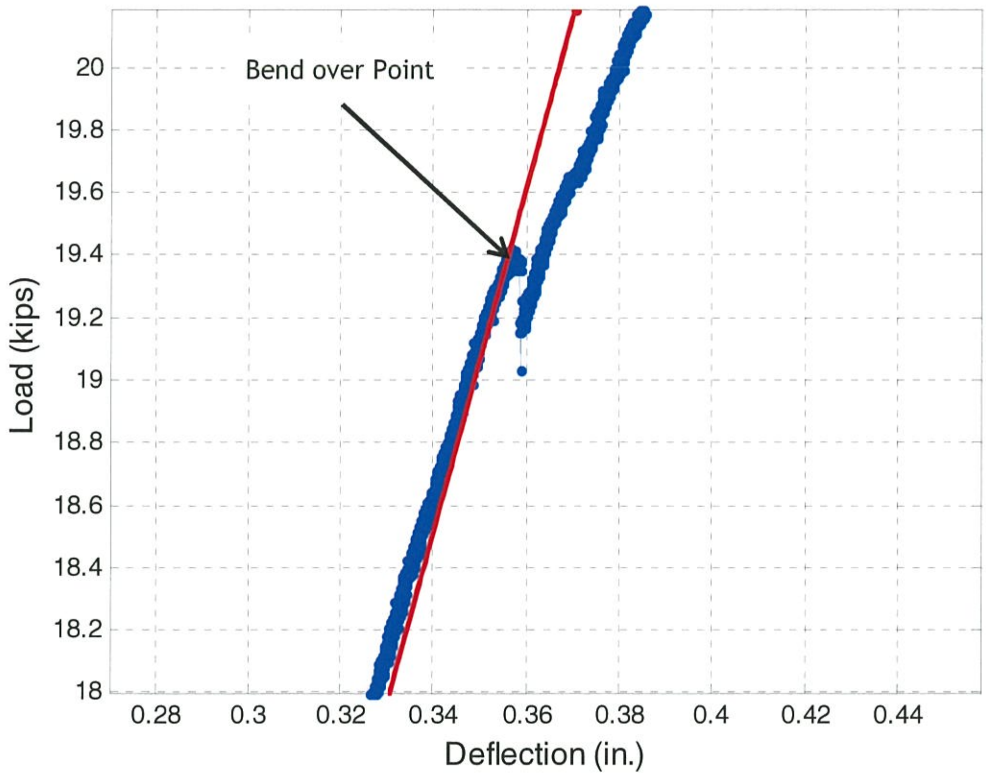

Figure 15: Bend Over Point to Determine Cracking

Errors in Predicted versus Measured Behavior

The errors between the predicted and measured cracking load, ultimate load, and maximum deflection were calculated to be 10.91%, 4.9%, and 11.14%, respectively. The total error from these three values was 26.96%. Table 4 below summarizes the designed values, predicted values, and actual values for these quantities. As stated previously, the greatest reason for the discrepancies between the designed and predicted strengths is the smaller eccentricity of the strands due to a construction error in the as-built specimen.

Table 4: Summary of Strength and Deflection Values

	Competition Requirement	Designed	Predicted	Actual
Cracking Strength (kips)	18.75	20	17.5	19.41
Ultimate Strength (kips)	30-37	33.5	31.2	32.73
Deflection (inches)	-	-	2.307	2.564

Recommendations

Overall, the University of Notre Dame Big Beam team is satisfied with the performance of the beam and learned valuable lessons about prestressed concrete design, construction, analysis, and high-strength mix design that they will apply to their future career ambitions.The performance of the beam was consistent with its design and pre-test predictions, but there was room for improvement in the design and construction processes as follows:

1) Importantly, there was an error in the actual placement (i.e., eccentricity) of the strands which should have been avoided. This resulted in smaller cracking and ultimate strengths for the as-built beam as compared to the target design strengths. Because the design was conducted using conservative selections, the tested beam still satisfied all competition requirements. This taught the team not only about the importance of construction and the care that needs to be given to it, but also the reasons and benefits of building conservative choices into the design.

2) Another complication that the team faced occurred during casting day. Despite ensuring proper consistency of the concrete mixture in laboratory trials, there were issues with segregation of mortar and aggregates during the casting of the test beam using ready-mix concrete. This provided a learning experience for future construction projects. The team now understands the importance of further mixing of concrete after deposition and also the difficulties of high-strength concrete design. As a result of the segregation, the compressive strength of the concrete in the as-built beam was about 2,000 psi smaller than that of the laboratory mix which is further detailed in the Concrete Mix Design section.

3) Due to a misunderstanding between the design and construction teams, the formwork was built to be 2.75 inches too tall. This issue was resolved by inserting a block of foam at the base of the formwork, but could have been prevented by a more collaborative design and construction approach amongst the team members. This was again a learning experience for the team in building their awareness of potential construction errors and approaches to solve them.

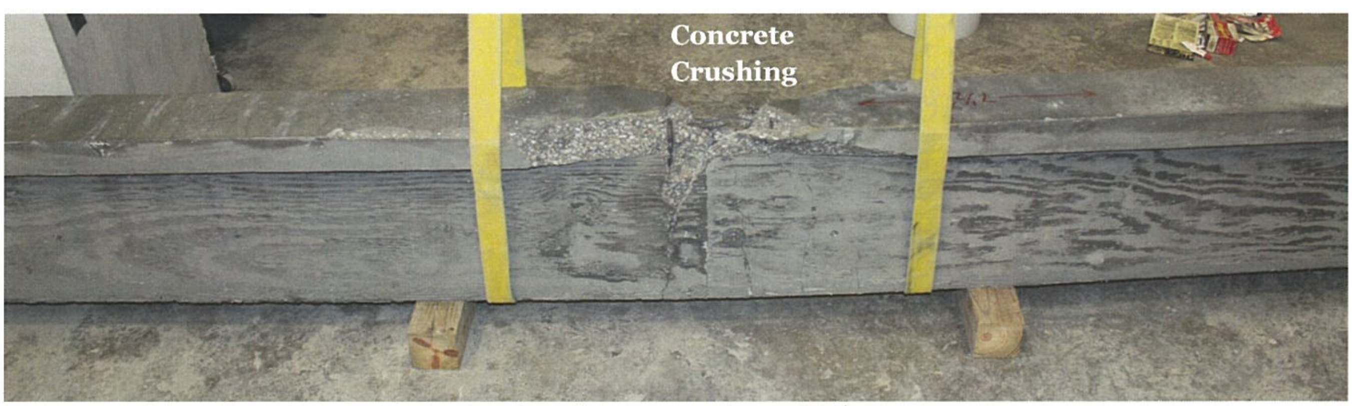

Figure 16: Beam at Failure

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Appendices

Appendix A: General Beam Information

A.1 Design Material Properties

Design Compressive Strength of Concrete	$f_c' = 17\ ksi$
Design Tensile Strength of Concrete	$f_r = 7.5\sqrt{f_c'} = 0.978\ ksi$
Ultimate Strength of Prestressing Strand	$f_{pu} = 270\ ksi$
Prestressing Strand Initial Stress	$f_{ps} = 0.7 * f_{pu} = 189\ ksi$
Total Area of Prestressing Strand	$A_{ps} = 3 * 0.153 = 0.459\ in^2$
Total Area Top Mild Steel	$A_s = 0.22\ in^2$
Unit Weight of Concrete	$\gamma = 150\ pcf$
Concrete Modulus of Elasticity	$E_c = 57{,}000 * \sqrt{f_c'} = 7{,}753\ ksi$
Prestressing Steel Modulus of Elasticity	$E_{ps} = 28{,}500\ ksi$

A.2 Geometric Properties

Beam Height	$h = 15\ in.$
Flange Height	$h_f = 2.75\ in.$
Flange Width	$b_f = 6\ in.$
Web Width	$b_w = 3\ in.$
Span Length	$L = 15\ ft.$
Total Beam Length	$L_t = 16\ ft. 4\ in.$
Moment of Inertia	$I = 1110.5\ in^4$
Distance to Centroid from Top Fiber	$c_1 = 6.55\ in.$
Distance to Centroid from Bottom Fiber	$c_2 = 8.45\ in.$
Top Section Modulus	$S_1 = \frac{I}{c_1} = 169.5\ in^3$
Bottom Section Modulus	$S_2 = \frac{I}{c_2} = 131.4\ in^3$

A.3 Cost Calculation

Cross Sectional Area	$A = 53.25\ in^2$
Formwork Surface Area	$SA = 49.74\ in^2$
Volume of Beam	$V = 0.205\ yd^3 = 5.55\ ft^3$
Weight of Beam	$W = 832.5\ lbs$
Weight of Mild Steel	$W_{MS} = 31.2\ lbs$
Cost of Concrete	$C_C = 120\left(\frac{\$}{yd^3}\right) * V(yd^3) = \$24.65$
Cost of Prestressing Steel	$C_{PS} = 0.30\left(\frac{\$}{ft.}\right) * 3 * L_t(ft.) = \$14.70$
Cost of Mild Steel	$C_{MS} = 0.45\left(\frac{\$}{lb.}\right) * W_{MS}(lb.) = \$14.04$
Cost of Formwork	$C_f = 1.25\left(\frac{\$}{ft.^2}\right) * SA(ft.^2) = \$62.17$
Total Cost of Beam	$C = \$115.63$

Appendix B: Design Calculations

B.1 Allowable Stress Design

Cement Factor	$\alpha = 4.0$
Type I Cement Moist Cured Factor	$\beta = 0.85$
Transfer Time	$t_{transfer} = 3\ days$
3 Day Test Concrete Strength	$f'_{c3} = 10.2\ ksi$
Initial Tension Stress Limit	$\sigma_{ti} = 3\sqrt{f_{ci} * 1000} = 0.30\ ksi$
Initial Compression Stress Limit	$\sigma_{ci} = -0.7 * f_{ci} = -7.00\ ksi$
Service Uncracked Tension Stress Limit	$\sigma_{ts} = f_r = 0.978\ ksi$
Service Compression Stress Limit	$\sigma_{cs} = -0.6 * f'_c = -10.2\ ksi$

B.1.1 Magnel Diagram Calculations

Initial Prestressing Force	$P_i = A_{ps} * f_{ps} = 86.75\ kips$
Prestressing Force After Losses	$P_e = 0.925 * P_i = 80.24\ kips$
Beam Self-Weight Load	$w_o = \gamma * \frac{A}{144} = 55.47 \frac{lb}{ft}$
Beam Self-Weight Moment	$M_o = \frac{w_o * L^2}{8000} = 18.72\ kip - in.$
Service Load	$P_s = 20\ kips$
Beam Service Moment	$M_s = \frac{P_s * 90in.}{2} + M_o = 918.7\ kip - in = 76.56\ kip - ft$

B.1.2 Magnel Equations

1. Eccentricity Associated with Tensile Stress at Top Fiber at Prestressing

$$e_1 = \frac{S_1}{A} + \frac{\sigma_{ti} * S_1}{P_i}$$

2. Eccentricity Associated with Compressive Stress at Bottom Fiber at Prestressing

$$e_2 = -\frac{S_2}{A} + \frac{-\sigma_{ci} * S_2}{P_i}$$

3. Eccentricity Associated with Compressive Stress at Top Fiber at Service Load

$$e_3 = \frac{S_1}{A} + \frac{\sigma_{cs} * S_1 + M_s}{P_e}$$

4. Eccentricity Associated with Tensile Stress at Bottom Fiber at Service Load

$$e_4 = -\frac{S_2}{A} + \frac{-\sigma_{ts} * S_2 + M_s}{P_e}$$

5. Eccentricity Controlled by Geometry of the Beam

$$e_{max} = c_2 - 1.75 = 6.70\ in.$$

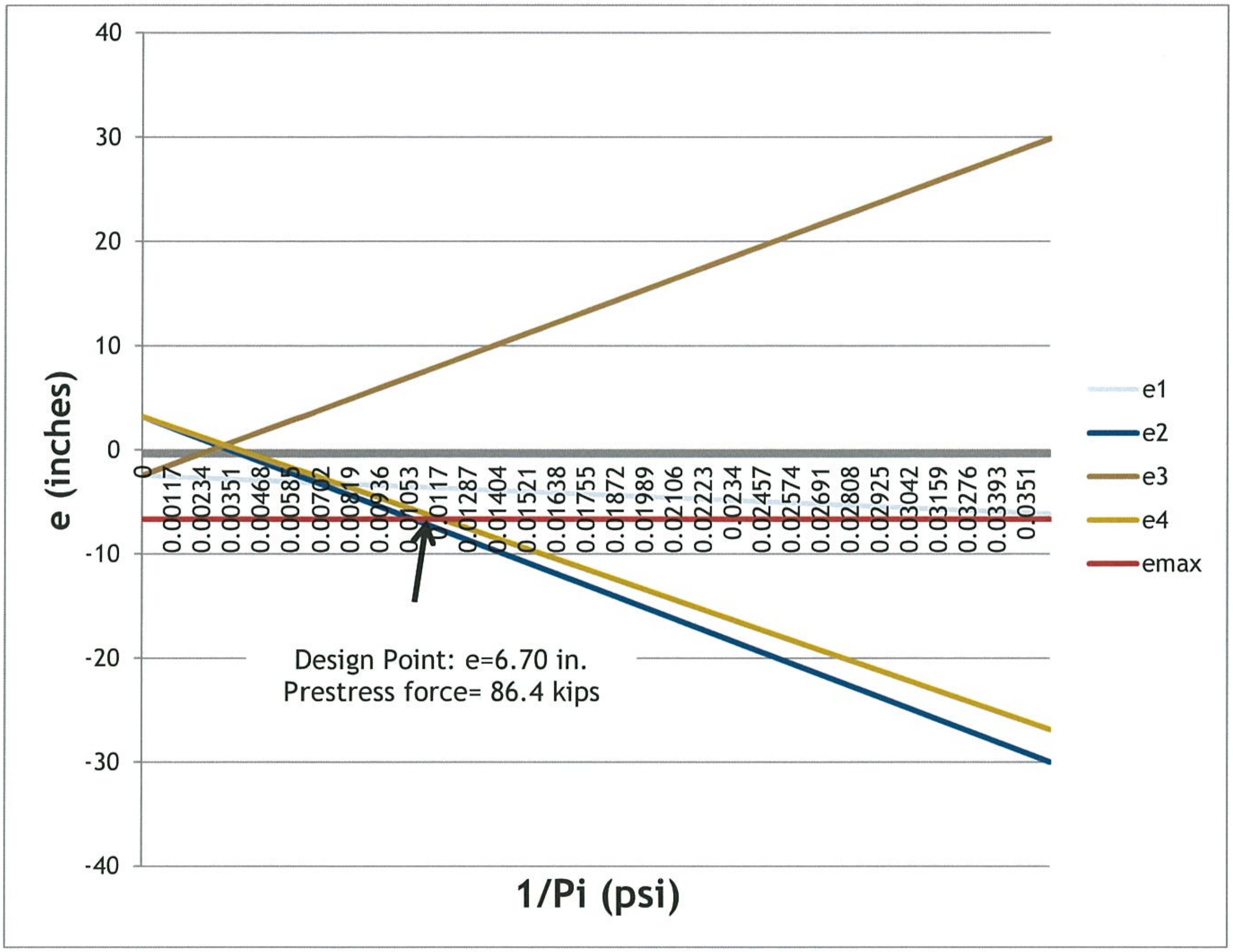

Figure B1: Magnel Diagram (e_1 ignored by design of reinforcement in flange)

B.2 Ultimate Flexural Strength Design

1. *Determine Moment Demand*

Maximum Load $P_u = 33.5\ kips$

Maximum Moment $M_u = \frac{P_u * 90in.}{2} + M_o = 1526.2\ kip - in.$

2. *Determine Compression Zone Resultant in Flange*

Depth of Prestressing $d_p = h - 1.75\ in. = 13.25\ in.$

Compressive Force from Concrete

$$\bar{C} = \frac{M_n}{d_p - \frac{a}{2}} = 121.6\ kips\ (found\ by\ iterating\ on\ a)$$

Depth of Compression Zone

$$a = \frac{\bar{C}}{0.85 * f_c' * b_f} = 1.40\ in. (found\ by\ iterating\ on\ \bar{C})$$

3. *Determine Location of Neutral Axis*

Beta 1 Coefficient $\beta_1 = 0.65$

Neutral Axis Depth $c = \frac{a}{\beta_1} = 2.16\ in.$

4. *Determine Stress of Prestressing Steel*

Ultimate Strain of Concrete $\epsilon_{cu} = 0.003$

Radius of Gyration Squared $r^2 = \frac{I}{A} = 20.85\ in^2$

Decompression Force $F_d = P_e\left[1 + \frac{E_p A_p}{E_c(A - A_p)}\left(1 + \frac{e^2}{r^2}\right)\right] = 26.84\ kips$

Prestrain of Prestressing Strand $Prestrain = \frac{F_d}{A_p E_p} = 0.002$

Ultimate Strain of Strand $\epsilon_{ps} = \epsilon_{cu}\frac{(d_p - c)}{c} + Prestrain = 0.00174$

Stress of Prestressing Strand

$$f_{ps} = 262.5\ ksi\ Using\ Actual\ f_{ps} vs.\ \epsilon_{ps}\ Relationship$$

5. *Determine if Mild Steel is Necessary*

Tensile Force of Strand $T_p = A_p f_{ps} = 120.5\ ksi$

Tensile Force of Mild Steel $T_s = A_s f_y$

Satisfy Equilibrium $T_p + T_s = \bar{C} \rightarrow T_p \approx \bar{C}\ so\ A_s = 0.018\ in^2$

$\therefore A_s\ is\ insignificant\ \therefore No\ Mild\ Steel\ Needed$

B.2.1 Design of Longitudinal Rebar to Resist Prestress Transfer

1. *Determine Uncracked Transformed Area and Moment of Inertia*

Assume Flange Longitudinal Steel as 2 No. 3 Bars

Area of Flange Longitudinal Steel	$A_s = 0.22\ in^2$
Modular Ratio	$n = \frac{E_s}{E_C} = 3.83 \rightarrow Round\ up\ to\ 4$
Uncracked Transformed Area	$A_{ut} = A + A_s(n-1) + A_{ps}(n-1) = 55.3\ in^2$
Area of Flange	$A_f = h_f b_f = 16.5\ in^2$
Centroid of Flange from Top	$y_f = \frac{h_f}{2} = 1.375\ in$
Area of Web	$A_w = (h - h_f)b_w = 36.75\ in^2$
Centroid of Web from Top	$y_w = \frac{h+h_f}{2} = 8.875\ in.$

Centroid of Flange Longitudinal Steel

$$y_s = Cover + Stirrup\ Diameter + \frac{Bar\ Diameter}{2}$$

$$y_s = \frac{3}{8}in + \frac{3}{8}in + \frac{3}{16}in = 0.9375\ in$$

Centroid of Uncracked Transformed Section	$\bar{Y} = \frac{A_f y_f + A_w y_w + A_s y_s(n-1) + A_{ps} d_p(n-1)}{A + A_s + A_{ps}} = 6.82\ in.$
Moment of Inertia of Flange about Centroid	$I_f = \frac{b_f h_f^3}{12} = 10.4\ in^4$
Moment of Inertia of Web about Centroid	$I_w = \frac{b_w (h-h_f)^3}{12} = 459.6\ in^4$

Moment of Inertia of Uncracked Transformed Section

$$I_{ut} = I_f + A_f(y_f - \bar{Y})^2 + I_w + A_w(y_w - \bar{Y})^2 + A_s(y_s - \bar{Y})^2(n-1) + A_{ps}(y_{ps} - \bar{Y})^2(n-1)$$

$$I_{ut} = 1194.3 in^4$$

2. Determine Location of Neutral Axis

Tensile Stress at Top Fiber at Prestressing $f_t = -\frac{P_i}{A} + \frac{P_i e_{max} c_1}{I_{ut}} + \frac{M_o c_1}{I_{ut}} = 1.51\ ksi$

Location of Neutral Axis Above c1 $f_t = 0 = -\frac{P_i}{A} + \frac{P_i \overline{y'} c_1}{I_{ut}} + \frac{M_o \overline{y'}}{I_{ut}} \therefore \overline{y'} = 3.33\ in.$

Location of Neutral Axis from Top $\bar{y} = c_1 - \overline{y'} = 3.21\ in.$

(Assume Triangular Stress Distribution)

Stress at Bottom of Flange $f_o = \frac{f_t}{\bar{y}}(\bar{y} - h_f) = 0.22\ ksi$

Tensile Force of Tensile Zone $T = \frac{b_w f_o(\bar{y} - h_f)}{2} + \frac{f_o + f_t}{2}(h_f - \bar{y})b_f = 14.4\ kips$

Area of Steel Required $A_s = \frac{T}{f_y} = 0.24\ in^2 \approx 0.22\ in^2$

$$\therefore 2\ No.3\ Bars\ Acceptable$$

B.3 Shear Design

1. Determine Shear Demand across the Beam

Shear from Support $V_{support} = \frac{P_u \times (1000\frac{lb}{Kip})}{2} = 16{,}750\ lb$

Location along Beam (in.) x

Shear from Self Weight (lb.) $V_{sw} = \frac{w_o}{(12\frac{in}{ft})}\left(\frac{L \times (\frac{12in}{ft})}{2} - x\right)$

Shear Demand (lb.) $V_u = V_{support} + V_{sw}$

2. Determine Shear Capacity of Concrete and Its Minimum and Maximum Values

Moment Demand (lb.-in.) $M_u = (V_u \cdot x) - (\frac{w_o \times \left(\frac{1ft}{12in}\right)}{2})(x^2)$

Strength Reduction Factor for Shear $\Phi = .75$

Concrete Shear Capacity (lb.) $V_c = Least\ of\ (a), (b), and\ (c)$

$$(a)\ V_c = (.6\sqrt{f_c'(\frac{1000lb}{Kip})} + 700\frac{V_u \times d_p}{M_u})b_w d_p$$

$$(b)\ V_c = (.6\sqrt{f_c'(\frac{1000lb}{Kip})} + 700)b_w d_p$$

$$(c)\ V_c = 5\sqrt{f_c'(\frac{1000lb}{Kip})}b_w d_p$$

Minimum Concrete Shear Capacity $V_{cmin} = 2\sqrt{f_c'(\frac{1000lb}{Kip})}b_w d_p = 10{,}365.53\ lb.$

Maximum Concrete Shear Capacity $V_{cmax} = V_{cw} = 3.5\sqrt{f_c'\left(\frac{1000lb}{Kip}\right)}\ b_w d_p = 25{,}913.83\ lb.$

3. *Determine Nominal Shear Strength of Shear Reinforcement*

Reinforcement Shear Capacity (lb.) $V_s = \frac{V_u}{\Phi} - V_{cgovern}$

Maximum Reinforcement Shear Capacity $V_{smax} = 8\sqrt{f_c'(\frac{1000lb}{Kip})}b_w d_p = 41{,}462.13\ lb.$

4. *Determine Minimum and Maximum Stirrup Spacing*

Area of Steel Stirrup $A_v = 0.11\ in.^2$ (assumed one No. 3 Bar)
Stirrup Steel Yield Strength $f_{yt} = 60{,}000\ psi$
Minimum Spacing (in.) $s_{min} = \frac{A_v f_{yt} d_p}{V_s}$
Maximum Spacing $s_{max} = 0.75 \times h = 11.25\ in.$
Governing Spacing: $S_{govern} = s_{max}\ if\ s_{min} > s_{max}\ else\ S_{govern} = s_{min}$

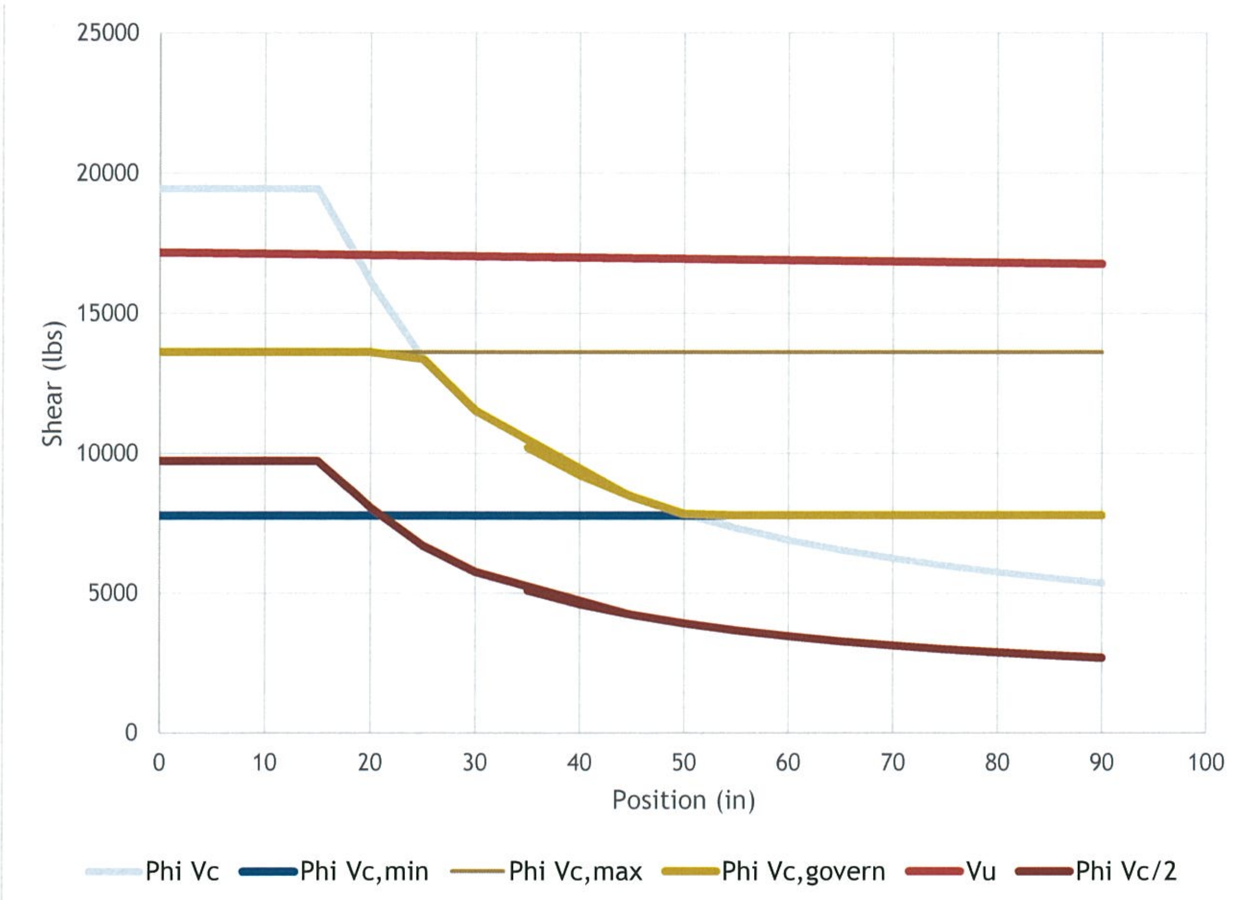

Figure B2. Shear Demand and Capacity over Half of Beam Span Length

Appendix C: Pretest Prediction Calculations

C.1 Cracking Moment Strength Load

Experimental Concrete Tensile Strength $f_r = 1.2735\ ksi$

Cracking Moment $M_{cr} = f_r S_b + P_e\left(\frac{r^2}{c_b} + e\right) = 67.17\ kip - ft$

Associated Load $P_{cr} = \left(\frac{M_{cr} - M_o}{L}\right) = 17.5\ kips$

C.2 Ultimate Strength Load

Assumed Stress in Prestressing Strands $f_{pf} = 0.9 f_{pu} = 243\ ksi$

Force in Prestressing Strands $T_p = A_p f_{pf} = 111.5\ kips$

Tension Force in Top Mild Steel $T_s = 0$

Compression Force in Top Steel $C_s = 0.22 f_y = 13.2\ kips$

Concrete Force $\bar{C} = T_p + T_s - C_s = 98.3\ kips$

Compression Block Depth $a = \frac{\bar{C}}{0.85 f_c' b_f} = 1.29\ inches$

Neutral Axis Depth $c = \frac{a}{\beta} = 1.98\ inches$

Prestrain $\epsilon_{pre} = \frac{F_d}{A_p E_p} = 0.0062$

Where: $F_d = P_e\left[1 + \frac{E_c}{E_p}\frac{A_p}{A_{conc}}\left(1 + \frac{e^2}{r^2}\right)\right] = 80.45\ kips$

Prestressing Strand Strain $\epsilon_{pf} = \frac{\epsilon_{cu}(d_p - c)}{c} + \epsilon_{pre} = 0.0217$ ***Iterated on in Excel**

Final Prestressing Strand Strain $\epsilon_{pf} = 0.0196$

Final Stress in Prestressing Strands $f_{pf} = 267.52\ ksi$

Nominal Moment Strength of Strands $M_n = T_p\left(d_p - \frac{a}{2}\right) + C_s\left(0.75 - \frac{a}{2}\right) = 118.57\ k\ ft.$

Load Associated with Nominal Moment $P_n = \frac{M_n - M_o}{L} = 31.2\ kips$

C.3 Deflection

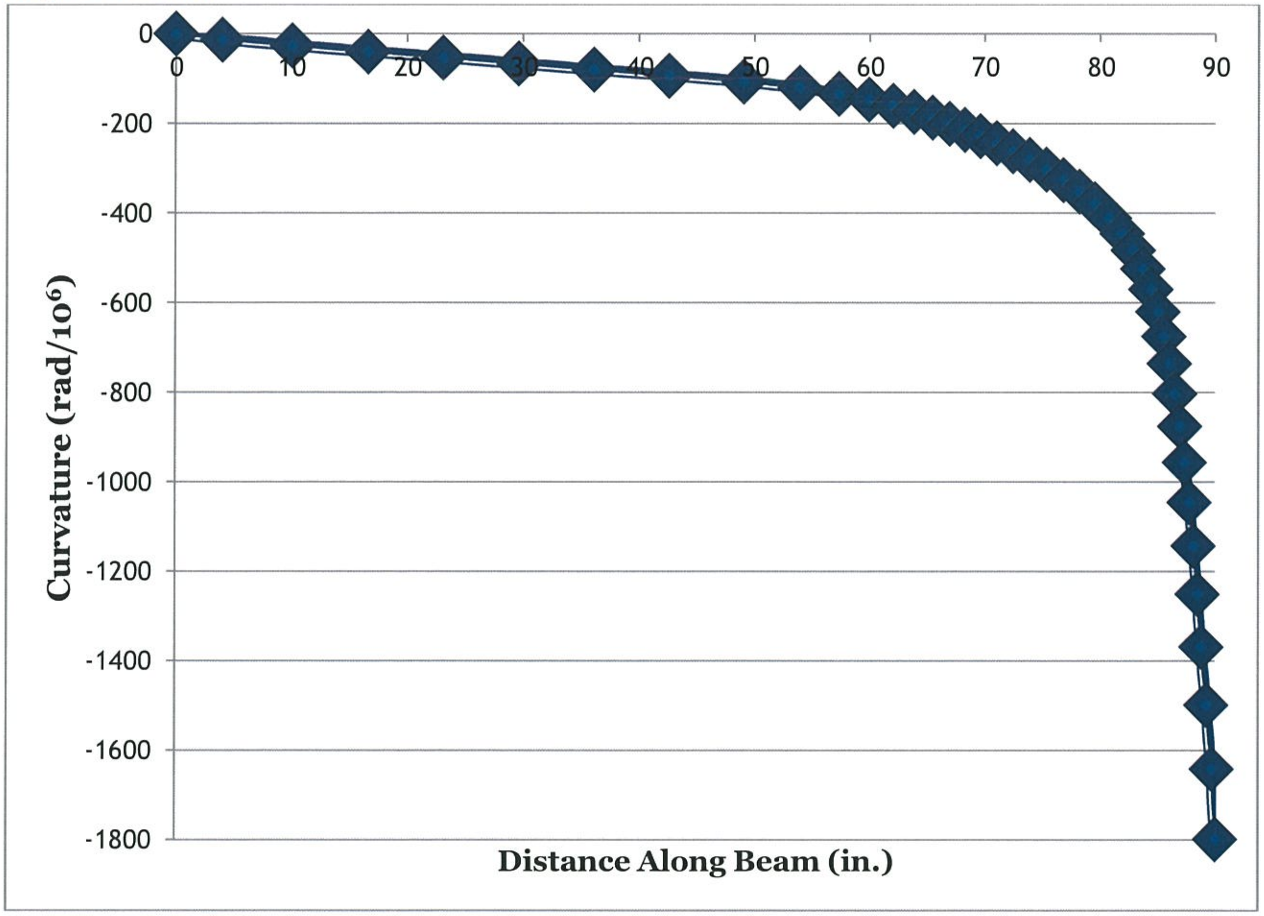

Figure C1. Predicted Curvature over Half of Beam Span Length

Table C.1 Prediction of Maximum Midspan Deflection

Distance (in.)	Moment (kips-ft)	Curvature (rad)	Area (in^2)	Centroid to Beam Midspan	Delta
0	0	0	0.000027	87.3117	0.002347
4.032	5.046	0.000013	0.000121	82.6183	0.009962
10.061	12.59	0.000027	0.000217	76.4692	0.016583
16.567	20.731	0.000040	0.000304	70.0258	0.021257
23.072	28.871	0.000053	0.000390	63.5561	0.024800
29.575	37.009	0.000067	0.000477	57.0755	0.027213
36.077	45.145	0.000080	0.000563	50.5901	0.028496
42.576	53.278	0.000093	0.000649	44.1088	0.028608
49.062	61.394	0.000107	0.000550	38.4622	0.021169
53.918	67.471	0.000120	0.000429	34.3594	0.014733
57.303	71.707	0.000133	0.000364	31.3761	0.011419
59.903	74.96	0.000147	0.000324	29.0240	0.009414
62.018	77.607	0.000160	0.000302	27.0646	0.008165
63.828	79.872	0.000173	0.000289	25.3586	0.007332
65.435	81.882	0.000187	0.000282	23.8282	0.006715
66.892	83.706	0.000200	0.000279	22.4260	0.006252
68.241	85.394	0.000213	0.000301	21.0697	0.006336
69.605	87.1	0.000228	0.000328	19.6912	0.006469
70.997	88.843	0.000244	0.000360	18.2815	0.006584
72.423	90.627	0.000261	0.000392	16.8447	0.006608
73.870	92.438	0.000281	0.000424	15.3937	0.006523
75.325	94.258	0.000302	0.000453	13.9439	0.006319
76.770	96.066	0.000325	0.000474	12.5203	0.005939
78.172	97.821	0.000351	0.000488	11.1517	0.005439
79.507	99.492	0.000379	0.000487	9.8680	0.004806
80.740	101.035	0.000411	0.000475	8.6967	0.004131
81.851	102.425	0.000445	0.000450	7.6571	0.003444
82.821	103.639	0.000482	0.000419	6.7570	0.002828
83.653	104.68	0.000524	0.000387	5.9878	0.002317
84.361	105.566	0.000569	0.000362	5.3303	0.001927
84.969	106.327	0.000620	0.000343	4.7619	0.001633
85.499	106.99	0.000675	0.000340	4.2559	0.001448
85.982	107.594	0.000735	0.000350	3.7870	0.001326
86.437	108.164	0.000802	0.000372	3.3375	0.001242
86.881	108.719	0.000875	0.000397	2.8989	0.001152
87.315	109.262	0.000956	0.000401	2.4819	0.000994
87.715	109.763	0.001045	0.000406	2.0962	0.000852
88.087	110.228	0.001143	0.000419	1.7355	0.000727
88.437	110.666	0.001250	0.000437	1.3936	0.000609
88.771	111.084	0.001368	0.000647	0.9999	0.000647
89.222	111.649	0.001498	0.000715	0.5463	0.000391
89.678	112.219	0.001641	0.000554	0.1586	0.000088
90	112.622	0.001799			
		SUM:	0.016746991	**SUM:**	0.325243494

Total Deflection:	1.1820	in.
Camber:	1.125	in.
Deflection of Beam:	2.307	in.

C.4 Percent Error of Predictions

Predicted Maximum Applied Load $P_{np} = 31.2\ kips$

Actual Maximum Applied Load $P_{na} = 32.73\ kips$

Maximum Applied Load Error $E_{pn} = \frac{|P_{np} - P_{na}|}{P_{np}} = 0.04904$

Predicted Cracking Load $P_{crp} = 17.5\ kips$

Actual Cracking Load $P_{cra} = 19.41\ kips$

Cracking Load Error $E_{P_{cr}} = \frac{|P_{crp} - P_{cra}|}{P_{crp}} = 0.10914$

Predicted Maximum Deflection $\Delta_{maxp} = 2.307\ in$

Actual Maximum Deflection $\Delta_{maxa} = 2.564\ in$

Maximum Deflection Error $E_{\Delta max} = \frac{|\Delta_{maxp} - \Delta_{maxa}|}{\Delta_{maxp}} = 0.11140$

Total Prediction Error $E_{tot} = E_{pn} + E_{P_{cr}} + E_{\Delta max} = 0.2696$